Chaos-Based Asynchronous DS-CDMA Systems and Enhanced Rake Receivers: Measuring the Improvements

Gianluca Mazzini, Riccardo Rovatti, and Gianluca Setti

Abstract—Results of recent theoretical investigations highlighted that the use of chaos in direct-sequence code-division multiple access (DS-CDMA) systems may lead to nonnegligible improvements in communication quality for several scenarios. We here briefly review the main steps in this derivation and report the experimental verifications of the corresponding theoretical prediction. With this, we confirm that chaos-based spreading sequences outperform classical pseudo-random sequences in at least two important cases. Over nonselective channels, the ability of chaos-based spreading of minimizing multiple-access interference leads to a measured 60% improvement in $P_{\text{err}}$ with respect to classical spreading. Over selective channels, the possibility of jointly optimizing chaos-based spreading and rake receiver profiles leads to improvements of up to 38% in $P_{\text{err}}$ with respect to systems with either conventional spreading or conventional rake policies.

Index Terms—Chaos, code-division multiple access, communication-system performance, rake receiver.

I. INTRODUCTION

WITHIN the past decade, it has become possible to directly approach the study of chaotic systems and to effectively exploit their potential for improving performance, with respect to classical solutions, in several signal processing tasks. The main reason for such an event is surely linked to the awareness that chaotic systems possesses a nature at the borderline between deterministic and stochastic, and to the consequent adoption of tools from statistical dynamical systems theory for their analysis [1].

The adoption of these statistical methodologies helps clarifying that the application fields which are most likely to benefit from chaos-based techniques are those where the statistical properties of the signals are the dominant factor. Additionally, the same tools have also been the key factor in developing the quantitative models needed to effectively design nonlinear systems obeying engineering specifications.

Within this framework, spread-spectrum communication has been thoroughly investigated by highlighting the trade-offs that chaos-based techniques may help to address as well as several foreseeable advantages in their applications.

The idea of using chaos in this filed is dated back to [2]–[5], where the possibility of generating an infinite number of spreading sequences for a standard DS-CDMA system by means of chaotic time series is first claimed. In [6] and [7], a first model of the performance of a chaos-based DS-CDMA system is reported leading to encouraging analytical and numerical results. These contributions succeeded in demonstrating the potentials of chaos-based techniques in this field that have been further developed in [8]–[11], where the impact of the adoption of chaos-based sequences in a standard asynchronous DS-CDMA systems is studied when additive white Gaussian noise (AWGN) channels are considered and multiuser interference (MAI) is the dominant cause of nonideality. For such environment, theoretical performance bounds have been obtained which show that significant improvement can be achieved by employing a properly designed family of chaotic systems generating real trajectories that are then quantized and periodically repeated to yield the users signatures. This result has been further strengthened in [12] where it is shown that chaos-based spreading allows to practically reach the absolute minimum of MAI. As such, a lower bound is approximately 15% lower than the average interference achievable by classical pseudo-random sequences, a definite improvement is demonstrated.

Though the users are not synchronized one with the other, synchronization between the transmitter and the receiver for the same user must be guaranteed before a reliable communication link can be established. Few steps in the direction of analyzing this synchronization mechanism have been taken in [13], where it is found that, even if synchronization can always be ensured, sequence correlation properties affect the speed and the reliability of this mechanism so that chaos-based spreading may, again, offer advantages over classical sequences.

Some dispersive channels, i.e., channels in which reflection, refraction and diffraction phenomena are nonnegligible, have also been investigated in connection with normal integrate-and-dump receivers [22], [14]. Furthermore, the case of the more complex rake receiver has been investigated. In this setting, a FIR filter is put before a classical receiver, whose taps can be adapted on-line to follow the channel dispersion characteristics which are identified in parallel. A general method has been devised to adapt both sequences and receivers and optimize performance. Though in dispersive channels this method is approximate, theory predicts that it still results in better performance with respect to the conventional

Manuscript received March 2001; revised August 27, 2001. This work was supported in part by the European Community under the ESPRIT Project 31103 INSPECT. This paper was recommended by Guest Editor K. Yao.

G. Mazzini and G. Setti are with DI University of Ferrara, Ferrara 44100, Italy (e-mail: gmazzini@ing.unife.it; gsetti@ing.unife.it).
R. Rovatti is with DEIS University of Bologna, Bologna 40136, Italy (e-mail: rovatti@deis.unibo.it).
Publisher Item Identifier S 1057-7122(01)10379-X.
matched-filter policy usually adopted for rake-receivers [15], [24], [16]. Preliminary results on even more sophisticated architectures like parallel cancelers can be found in [17].

A good and fairly complete survey of the results in this field and in others related to chaos-based communication can be found in [18].

Beyond theoretical results, one of the first unavoidable steps toward the real-world exploitation of these innovative methodologies is surely the realization of a low-cost experimental set-up allowing field-validation. We here report on how these theoretical predictions have been tested by means of prototype measurements in the case of nondispersive channel and simple correlation receivers, as well as in complex multipath channels, where optimized rake receivers are employed to relieve selective fading effects. In order to correctly interpret these experimental results, we also give a tutorial overview of the theoretical background which is here presented using a slightly modified and more general framework with respect to previous publications.

The paper is organized as follows. In Section II, the channel and receiver models are illustrated to obtain a separate expression of all the components of the correlate-and-dump output before the hard decision block. These components are random variables whose sum is matched against a zero threshold to reconstruct the symbol. Their distribution affects the bit error probability, as considered in Section III. Following an established path the standard Gaussian approximation (SGA) is adopted to tackle the cross- and self-interference terms. On the contrary, SGA is not formally exact to model the mixed deterministic-random nature of the useful component, but is nevertheless able to take into account the two main factors determining the final performance in terms of bit error probability. Additionally, exploiting this approximation, it is possible to simply express system performance as a function of the channel parameters, of the spreading sequence statistical features and of the receiver FIR taps. In Section IV, such an expression is employed to state a simple constrained optimization problem in terms of the maximization of a cost function which combines multi-index quantities depending on the previous parameters. Such a problem is analytically solved to link the two design parameters (i.e., the spreading sequences and receiver FIR taps) to the channel characteristics. As a particular, but important case, it is shown that, when the channel is nondispersive, such an optimization procedure yields the same result presented in [12] for the spreading sequences allowing the minimization of MAI.

In Section V, the problem of generating spreading sequences with different correlation behaviors, and thus with different impact on the FIR optimization, is addressed. First, an established method for quantizing a slice of a chaotic trajectory generated by the iteration of a simple dynamical system is recalled. Then, the mathematical tools for the analytical evaluation of any-order partial correlation of those sequences are applied to give an expression to all the quantities that have to be known to design optimal chaos-based spreading sequences, when MAI is the unique cause of nonideality, and to jointly optimize sequences statistics and receiver FIR taps in case of selective channel. This result is the quantitative aspect of the general idea that the ability of controlling the correlation behavior of signals benefits the mechanisms that process them.

In Section VI details on a low-cost DSP-based DS-CDMA system implementation are shown, along with the measured bit error probability when classical pseudo-random and chaos-based sequences are employed for spectrum-spreading. Measured performance results to be in good agreement with the theoretical analysis. Finally, Section VII reports some conclusive remarks and highlight future research directions.

II. ASYNCHRONOUS DS-CDMA SYSTEMS MODEL

Fig. 1 reports the simplified baseband equivalent scheme of an asynchronous DS-CDMA system in which the carrier is common, $U$ users are supposed, and $\ell^u$ and $\theta^u$ indicate respectively the absolute delay and carrier phase of the generic $u$th user signal, the $u$th being the useful one. These last quantities are assumed as independent and uniformly distributed random variables to model transmission from mobile terminals to a fixed base-station.

As far as transmitted signals are concerned, the generic $u$th users information signal is

\[ S^u(t) = \sum_{s=-\infty}^{\infty} S^u_s g_T(t-sT), \]

where $g_T(t)$ is the rectangular pulse which is 1 within $[0,T]$ and vanishes otherwise. Additionally, we will assume that the information symbols $S^u_s = \pm 1$ are produced by perfectly uncorrelated sources, that are also independent one from the other.

The spreading signal depends on sequences of symbols $s^u_s$ from the alphabet $X$ which are mapped into $\{-1, +1\}$ by the function $Q$, and combined to form the complex signal

\[ Q^u(t) = \sum_{s=-\infty}^{\infty} Q(s^u_s) g_T(t-sT/N). \]

The signal $Q^u(t)$ is then multiplied by $S^u(t)$ and transmitted along the (equivalent) channel together with the spread-spectrum signals from the other users. Each user adopts a different spreading code, assigned at the connection start-up. Following the approach in [19], [8] we will assume that spreading sequences are periodic of a period equal to the spreading factor $N$.

We will model the selective fading communication channel as a linear time-varying propagation medium, where the multipath effect is due to the simultaneous presence of $\Delta + 1$ rays for each transmitted signal. The channel transmits the signal of the
generic \( \nu \)th user as described by the following low-pass impulse response

\[
h_v^{\nu}(t) = \beta_1 \delta(t) + \sum_{\tau=0}^{\Lambda} \beta_\tau \alpha_\tau^{\nu} \delta \left( t - \tau \frac{T}{N} \right)
\]

where \( \delta(t) \) is the Dirac’s generalized function, \( \beta_1 \) is the direct ray power attenuation, \( \beta_\tau \) is the instantaneous random power attenuation of the \( \tau \)th ray whose delay is \( \tau T/N \). As thoroughly discussed in [20], considering ray delays, multiple of the chip time leads to no loss of generality.

The random nature of the first and secondary rays, assumed independent from each other, is modeled by the complex random variables \( \alpha_\tau^{\nu} \) whose real and imaginary part are independent and Gaussian with zero mean and variance \( 1/2 \).

This channel model is the same used in [14] (with the exception of the truncation to \( \Lambda \) rays) and in [15], [24] since, although not the most general possible setting, it allows to ease analytical computations and to better focus the impact of the adoption of chaos-based spreading.

We indicate with \( K \) the Rice factor, i.e., the ratio between the power of the first ray and the remaining rays, such that \( \beta_1 + \beta_\tau = K \sum_{\tau=1}^{\Lambda} \beta_\tau \) with \( (\beta_1 + \beta_\tau) (K + 1)/K = 1 \). Furthermore, to consider what fraction of the main ray power is associated to random component we refer to a second similar quantity \( \alpha = \beta_2^{\nu}/\beta_1 \). The channel adds up the contributions of all the users so that the baseband receiver is presented with a signal which is the sum of the convolutions of the signal transmitted by each user and the corresponding channel impulse response. Furthermore, all the random variables describing the channel contributions will be considered independent.

To cope with the selective fading effect due to the multipath propagating channel it is common practice to employ a rake filter with the aim of collecting time-scattered power before the signal undergoes common despreading operation. More specifically, consider the \( \nu \)th users rake receiver shown on the right part of Fig. 1. The presence of a further FIR filter of coefficients \( \gamma_{-\Lambda}, \gamma_0, \ldots, \gamma_\Lambda \) implies that the channel is cascaded with a further linear system whose impulse response is

\[
h_v^{\nu}(t) = \gamma_{-\Lambda} \delta \left( t - \Lambda \frac{T}{N} \right) + \sum_{\tau=0}^{\Lambda} \gamma_{\tau} (\alpha_\tau^{\nu})^* \delta \left( t - (\Lambda - \tau) \frac{T}{N} \right)
\]

where \( * \) stands for complex conjugation.

With this, the equivalent channel from the \( \nu \)th transmitter to the \( \nu \)th receiver has the impulse response \([h_v^{\nu} \star h_v^{\nu}](t)\).

The output of the rake filter is fed into a multiplier which combines it with a synchronized replica of the spreading sequence of the \( \nu \)th user and of an integrate-and-dump stage in charge of recovering the information symbol by correlation. Even if the unavoidable influence of thermal noise can be neglected, symbol extraction is hampered by the presence of the delayed version of the useful signal and by the current and delayed signals transmitted by the other users, who acts as interferers for the useful one.

The key to estimate the performance of such a communication system is the knowledge of how much the nonuseful signals affect the receiver output, i.e., on how much they are correlated with the spreading sequence of the useful user.

As correlation with a fixed sequence is a linear operation, error causes add at the receiver output before the hard decision block \( \Upsilon_s^{\nu} \) of the useful \( \nu \)th user, so that it can be decomposed into three main terms

\[
\Upsilon_s^{\nu} = \Omega_s^{\nu} + \Xi_s^{\nu} + \Psi_s^{\nu}
\]

where

- \( \Omega_s^{\nu} \) main equivalent ray carrying the information to be retrieved;
- \( \Xi_s^{\nu} \) disturbance due to the secondary equivalent rays carrying the previous symbols of the useful user;
- \( \Psi_s^{\nu} \) the disturbance due to the primary and secondary equivalent rays carrying the information transmitted by the other users.

III. SYSTEM PERFORMANCE MERIT FIGURE

The above decomposition of the decision signal is the base for the estimation of the performance of the system.

Since we assume that the speed of variation of the channel parameters is of the same order of the speed of variation of the transmitted signal, expectations on both channel-related (i.e., delays and phases) and transmission-related (i.e., information symbols) random variables can be considered at the same time. These are the conditions under which the error probability \( P_{\text{err}} \) is the most sensible merit figure. Considering the threshold mechanism at the end of the receiver, we can expresses it as

\[
P_{\text{err}} = \Pr\{\Upsilon_s < 0 | S_s^{\nu} = +1\}.
\]

A careful consideration of the nature of the interfering signals reveals that \( \Xi_s^{\nu} \) is uncorrelated with \( \Omega_s^{\nu} \) and that \( \Omega_s^{\nu} \) and \( \Psi_s^{\nu} \) are also uncorrelated [15], [24], [16], so that the computation of the previous quantity rely on the availability of the probability distribution of each of the three random variable composing \( \Upsilon_s^{\nu} \). To compute, we mainly resort on the SGA, i.e., we assume that sums of independent random contributions are considered to be Gaussian random variables, characterized only by means of their average and variance.

A discussion of this kind of approximation can be found in [16]. We here exploit it to its ultimate consequences to derive that, if all the random variables involved in the decision process can be considered Gaussian then the bit error probability can be surely given an expression of the kind

\[
P_{\text{err}} = \frac{1}{2} \text{erfc}(\sqrt{\rho})
\]

for a suitably defined signal-to-interference ratio (SIR) \( \rho \).

\[
\rho = \frac{\mathbb{E}[|S_s^{\nu}|^2]}{2(\sigma_0^2 + \sigma_\Psi^2 + \sigma_\Xi^2)}.
\]  

In our case, we may rewrite (1) in terms of the statistical behavior of both the spreading sequences and the channel taps, as well as of the taps of the FIR filter in the receiver.

To do so, we define the usual partial autocorrelation function \( \Gamma_N,\tau(x^{\nu}, \bar{x}^\nu) = \Gamma_N,\tau(x^{\nu}, \bar{x}^\nu) = \)
\[ \sum_{k=-N}^{N} Q(x_k^\tau)Q(x_{k+\tau}^\cdot) \quad \text{for} \quad \tau = 0, 1, \ldots, N - 1 \quad \text{and} \quad 0 \text{ otherwise.} \]

With this, we set
\[
c'^{\tau} = \begin{cases} \sum_{r=-N}^{N-1} E_{Z^r, Z^\tau} 
& \left[ \Gamma_{N-\tau}(x^r, x^\tau) + \Re\left[ \Gamma_{N-\tau}(x^r, x^\tau) \Gamma_{N-\tau+1}(x^r, x^\tau) \right] \right] \\
& \frac{E_x \left[ \Gamma_{N-\tau}(x^r, x^\tau) + \Gamma_{N-\tau+1}(x^r, x^\tau) \right]}{2N^2} 
& \text{for} \quad \tau > 0 \\
& 1 
& \text{for} \quad \tau = 0 \\
& \frac{E_x \left[ \Gamma_{N-\tau}(x^r, x^\tau) \right]}{2N^2} 
\end{cases}
\]

and, in turn, the \((\lambda + 1) \times (\lambda + 1)\) square matrix \(\mathcal{A}\)
\[
\mathcal{A}_{j_1,j_2} = 
\begin{cases} 
\sum_{j=-1}^{\lambda} [(U - 1)c'] \\
& + (1 - \delta_{j_1,j_1})c''_{j_1,j_1} (\beta)^2, \quad \text{if} \quad j_1 = j_2 = -1, \ldots, \lambda \\
& 2c''_{j_1,j_2} |\beta_1| \beta_{j_2}, \quad \text{if} \quad j_1 \neq j_2 
\end{cases}
\]

where \(\delta_{.,.}\) is the usual Kronecker's symbol.

With this, if we define the vectors \(\beta = (\beta_1, \ldots, \beta_{\lambda+2}) = (\beta_{-1}, \beta_0, \ldots, \beta_\lambda)\) and \(\gamma = (\gamma_1, \ldots, \gamma_{\lambda+2}) = (\gamma_{-1}, \gamma_0, \ldots, \gamma_\lambda)\) we finally obtain
\[
\rho = \frac{1}{2} \left( \frac{\beta^T \gamma}{\gamma^T \mathcal{A} \gamma} \right)^2
\]

where \(\cdot^T\) denotes vector transposition.

IV. PERFORMANCE OPTIMIZATION IN DISPERSIVE AND NON-DISPERSIVE CHANNELS

Once that an expression for \(\rho\) is given in terms of the adjustable parameters we may think of optimizing the system performance by maximizing \(\rho\).

More in detail note that in (2) we have expressed \(\rho = \rho(\mathcal{A}(\beta), \beta, \gamma)\) identifying the three factors affecting the performance, i.e., the statistics of the spreading sequences \(\mathcal{A}\), the channel power profile \(\beta\) and the rake power profile \(\gamma\).

Note also that, according to the physical intuition behind this dependence, \(\rho(\mathcal{A}(\xi), \xi, \beta, \gamma) = \rho(\mathcal{A}(\beta), \beta, \gamma)\) for any \(\xi, \xi' \neq 0\), since channel and receiver gain cannot affect the SIR.

The maximization of \(\rho\) can be now simplified exploiting this double scale-invariance of \(\rho\) and setting, without any loss of generality, \(\beta^T \beta = 1\) and \(\beta^T \gamma = 1\). With this the numerator in the expression of \(\rho\) is fixed and we may maximize it solving the minimization problem
\[
\min_{\gamma} \quad \gamma^T \mathcal{A} \gamma \\
\text{s.t.} \quad \beta^T \gamma = 1
\]

whose solution gives the optimal receiver FIR taps \(\gamma\) [21]
\[
\gamma = \frac{\mathcal{A}^{-1} \beta}{\beta^T \mathcal{A} \beta}
\]

Note that the above solution relies on the possibility of extracting averages of the incoming signal and thus of estimating the transfer function of the channel. Methods for this do exist, are commonly assumed available in the classical rake receiver Literature, and we assume that the resulting estimation is reliable.

Note also that such an optimization depends on \(\mathcal{A}\) and thus on the statistical properties of the spreading sequences and this offer room for further optimization if one may design sequences generators with prescribed statistical features, as it is possible in the chaos-based setting described in the next section. Additionally such a remark allow us to remark about the significance of (3) in some particular but important settings.

When information about the correlation of the spreading sequences is not available, one may assume that all the disturbances are white and thus proceed by maximization of the magnitude of the useful component \(E_x [\Omega(x^\tau) = +1] = (1/2) \beta^T \gamma\) only. As, under these assumptions, the power gain of the receiver acts both on the useful component and on the white disturbance, maximization can be performed for a unit power gain receiver as in
\[
\max_{\gamma} \quad \beta^T \gamma \\
\text{s.t.} \quad \gamma^T \gamma = 1
\]

which, when \(\beta^T \beta = 1\), has the trivial solution \(\hat{\gamma} = \beta\), as it would happen in a conventional rake receiver.

Consider now the much simpler case of a nondispersive channels, where \(\beta_\tau = 0\) for \(\tau \geq 0\) and \(\beta_{-1} = 1\). Hence the constraint \(\beta^T \gamma = 1\) sets \(\gamma_{\tau} = 0\) for \(\tau \geq 0\) and \(\gamma_{-1} = 1\) to avoid resorting to rake-receivers for a nondispersive channel. With this (2) is greatly simplified and \(\rho\) is maximized when the single quantity \(\mathcal{A}_{-1,-1} = (U - 1)c'\) is minimized.

Details of such a minimization in the more general context of complex spreading sequences can be found in [12] and [14]. It is here enough to recall that if all the sequences can be thought as independent and if they satisfy a mild form of second-order stationarity, namely
\[ E_x [Q(x^\tau)m^R(x^\mu)] = E_x [Q(x^\tau)Q^*(x^\mu)] \]
then, setting \(h = 2 - \sqrt{3}\) the previous expression reaches a minimum given by
\[
(U - 1) \frac{\sqrt{3}}{3N} \frac{h^{2N} - h^{2N}}{h^{2N} + h^{2N} - 2}
\]
when the sequence are characterized by a real, almost-exponential, sign-alternating auto-correlation profile given by
\[
E_x [Q(x^\tau)Q^*(x^\mu)] \approx (-1)^{\tau} \frac{N}{N - \tau} \frac{h^{2N} - h^{2N-k}}{h^{2N} - h^{2N-k}}
\]

It is also important to notice that, since nondispersive channels attach no randomness to the useful component \(\Omega\), the approximation that identifies \(f_{\Omega} = (\beta_{-1}, \gamma_{-1})\) with a Gaussian density is unnecessary. Hence, under the only assumption of
Gaussian interference, we are able to identify the absolute optimum choice for spreading sequences. Since for any practical \( N \) (4) is approximately 15\% less than \( 2(U - 1)/(3N) \) (the classical merit figure for ideal random sequences), a definite capacity improvement is demonstrated.

V. CHAOS-BASED GENERATION OF OPTIMIZED SPREADING SEQUENCES

To generate spreading sequences with chaotic maps, let us set \( X = [0, 1] \) and consider a one-dimensional chaotic system \( x_{k+1} = M(x_k) \) which is iterated \( N - 1 \) times starting from an initial condition \( x_0 \) which is randomly and independently drawn for each users according to a uniform probability density. The trajectory is quantized by a function \( Q: [0, 1] \rightarrow \{-1, +1\} \).

Here, we assume that such a quantization is such that \( n \) is an even integer and that all the points in the interval \( X_j = [(j - 1)/n, j/n] \) are mapped by \( Q \) to the same symbol.

We also refer to a particular family of maps which has been thoroughly characterized in [14, 11], namely the so called \((n, t)\)-tailed shifts defined as

\[
M(x) = \begin{cases} 
(n - t)x \left( \text{mod} \frac{n - t}{n} \right) + \frac{t}{n}, & \text{if } 0 \leq x < \frac{n - t}{n} \\
(t \left( x - \frac{n - t}{n} \right) \left( \text{mod} \frac{t}{n} \right), & \text{otherwise}
\end{cases}
\]

for \( t < n/2 \). The family of \((n, t)\)-tailed shifts includes some of the maps already analyzed in the chaos-based DS-CDMA framework [8] like the \( n \)-way Bernoulli shift [which is a \((n, 0)\)-tailed shift] and the \( n \)-way tailed shift [which is a \((n, 1)\)-tailed shift]. Fig. 2 shows the graphics of \((10, 3)\)-tailed shift.

For these maps, the entries of the matrix \( A \) in (2) can be analytically computed relying on results reported in [11, 22, 23]. The key property used in those works is that \((n, t)\)-tailed shifts are piecewise-affine Markov maps, i.e., that the statistical features of their trajectories can be studied referring to an “equivalent” Markov chain with \( n \) states (one for each interval \( X_j = [(j - 1)/n, j/n] \)).

Those calculation show that the two degree of freedom \( n \) and \( t \) in the design of the map are actually only one degree of freedom \( r = -t/(n - t) \) that, from now on, will be used to parameterize the map choice.

With this, we know that for each \( r \) (and thus for each sequence generation mechanism) the optimization procedure sketched in Section IV gives us the best receiving filter and thus the lowest achievable \( P_{\text{err}} \). This information can be used to iteratively search for that \( r \) whose corresponding optimum filter results in the minimum \( P_{\text{err}} \) thus simultaneously optimizing sequence generation and receiver taps. We will call this system OCOR, i.e., optimal chaos with optimal rake.

Other options to test are optimal chaos with conventional rake (OCCR) and no chaos with optimal rake (NOCR) as well as no chaos with conventional rake (NCCR) where nonchaotic sequences are assumed to be a good approximation of the purely random case as, for example, \( m \)- or Gold codes (see, e.g., [8] and references therein).

In nondispersive channels, \((n, t)\)-tailed shift allow the absolute optimization of the optimal chaos with no rake (OCNR) case. In fact, for any given \( n \), we may chose \( t \) as the integer closest to \( nh/(1 + h) \), the accuracy being obviously improved as \( n \rightarrow \infty \). With this the auto-correlation profile of the generated sequences matches (5) with a negligible error in practically any operation conditions.
VI. CHAOS-BASED DS-CDMA SYSTEM PROTOTYPE AND MEASUREMENT RESULT

With the aim of obtaining a first and effective confirmation of the theoretical results reported in the previous Sections, we realized a low-cost prototype of the communication system using TMS320C542 DSP boards. A picture of the whole system is reported in Fig. 3(a).

Fig. 3(b) reports the equivalent structure. Transmitter boards are in charge of generating baseband signals, of their BPSK modulation and of the simulation of the channel. To reduce computational burden the FIR simulating the channel is swapped with the BPSK modulation and operated at baseband level. No external RF circuit has been included in this low-cost implementation. One board is devoted to the implementation of the useful transmitter, one board is devoted to the implementation of the corresponding receiver and of the estimator of the bit error rate while all the other boards can be used to account for multiple access interference.

In Fig. 4 we report the detailed signal-level scheme of the computations carried out in each of the blocks in Fig. 3(b).

Spreading sequences are generated off-line and stored in the transmitters. A copy of the useful sequences is also stored in the receiver.

At symbol rate, information bits are locally generated. They are multiplied by the spreading sequence thus generating data at chip rate that are fed into a quadrature implementation of the five-taps (\(\Delta = 4\)) complex FIR simulating the channel.

The two baseband components are then multiplied by stored samples of \(\sin\) and \(\cos\) to generate the passband signals that are added and converted into the analog output of the transmitter. The offsets in the \(\sin\) and \(\cos\) table can be adjusted to account for different time-varying delays and carrier phases in the interfering transmitters, they are kept constant in the useful transmitter.

If \(B\) is the symbol rate, \(N \times B\) is the chip rate and, if \(M\) is the number of sinusoid samples per chip, \(M \times N \times B\) is the sample rate. Since the A/D and D/A conversion is in the audio band we get that the prototype transmits at \(B = 25 \times 10^3/(MN) \approx 6.5\) bits per second.

The same structure is replicated for the interfering transmitters in which a randomly variable delay and a random carrier phase are also introduced to account for the asynchronous environment. The delay and carrier phase is obtained in the digital chain by means of the adjustable offsets in the \(\sin\) and \(\cos\) table.

In the transmitters, digitally generated pseudo-random variables are used to generate information bits, generate and update real and imaginary parts of the channel FIR taps once every \(N_c\) bits, as well as to possibly generate and update interferer delays and carrier phases once every \(N_c\) bits.

The signal arriving at the receiver is converted into digital and fed into a quadrature demodulator relying again on \(\cos\) and \(\sin\) tables and on accumulators. This produces two streams of samples at chip-rate which are separately processed by a quadrature implementation of the rake FIR and finally summed.

The output of the rake is then fed into the correlation-based despreader and in the decision block.

A prerequisite for correct operation of the receiver is its synchronization with the transmitter along with the availability of a
perfect identification of the channel taps. In addition, to ascertain if the received bit is identical to the transmitted one the information stream generated at the transmitter must be also available. In fact, the two bit streams must be compared to compute the bit error rate.

As far as all the quantities depending on digital pseudo-random variables are concerned, we use a suitable digital generator addressing the trade-off between randomness and fixed-point computation whose seed is common to both the transmitter and receiver.

As far as the time-synchronization is concerned, the D/A converter of the useful transmitter and the A/D converter of the receiver share the same timing signals (a clock and a frame sync signal). At startup, a simple handshake is initiated relying on two more wires carrying interrupt signals from one board to the other [see Fig. 5(a)] to implement a classical request/acknowledge sequence.

Fig. 5(b) reports the tracks of the RQ, ACK signals, along with the output of the transmitter D/A and its synchronization signal FS.

Since the DSP clock frequency is 40 MHz and the time scale of the analog signals is within the \(\mu s\) range, digital events appear as impulses. Nevertheless, it is evident that from the very beginning the RQ signal is toggled by the transmitter to test receiver readiness.

Fig. 5. (a) Scheme indicating the signal involved in the handshake operation between transmitter and receiver for initial synchronization. (b) Corresponding oscilloscope diagrams.

After internal initialization, the receiver assesses its availability by pulsing the ACK signal. This starts the conversion in both the transmitter and receivers.

Note that a delay is present between the receiver acknowledge and the actual appearance of a meaningful signal on the transmission line. This is due to the smoothing filter at the output of the D/A and cannot be avoided. To cope with this the receiver is instructed to discard a certain number (determined experimentally) of samples actually postponing synchronization.

A. Performance Over Non-Dispersive Channel

The prototype system has been first exploited to test performance of CB-DS-CDMA systems when no multipath is present. In this case \(\Delta = 1, K = \alpha = \infty\).

Fig. 6(a) shows, from top to bottom, the signal at the output of the transmitter, of two different interferes and at the input of the receiver. Fig. 6(b) shows the effect of the random delay between the signals of the transmitter and of a interfering user.

Conventional rake and optimized rake coincide so that only two configurations must be tested, namely OCxx and NCxx. Table I compares the measured bit error rate when the prototype system is operated for \(2 \times 10^6\) bits for \(U = 6, N = 15\) and
N = 20, M = 256, and τi = 10. For every configuration performance is averaged over 100 sequence sets and considering the average $P_{err}$ experienced by every user.

Results confirm the fact that chaos-based DS-CDMA outperforms classical DS-CDMA. Also quantitative theoretical predictions are in good agreement with experimental results.

**B. Performance Over Exponential Channel**

To test performance over a dispersive channel we choose an exponential power profile characterized by $K = 1, \alpha = 10, \Lambda = 4$ and $\beta_k \sim e^{-k}$.

We used all the available interfering boards for a total of $U = 8$. We also have $N = 15, M = 256, \text{ and } \tau_c = \tau_i = 3$. The number of transmitted bits per configuration was $2 \times 10^8$.

For every configuration performance is averaged over 100 sequence sets. For every sequence set the average $P_{err}$ experienced by all the user is considered.

Again, results, reported in Table II confirm the fact that chaos-based DS-CDMA outperforms classical DS-CDMA and that the adoption of nonconventional and carefully designed receiver filters also contributes nonnegligibly to the quality of the communication.

**VII. Conclusion**

A prototype asynchronous DS-CDMA system was built compounding commercially available DSP boards. Their programmability gives the flexibility needed to try different spreading sequences as well as different scenarios.

As a first step, the prototype system was used to assess the real-world applicability of optimal spreading sequences whose existence is predicted by the theory when MAI is the dominant cause of transmission errors.

As a second step we tested the more sophisticated case in which a nonconventional adaptation policy is applied to a rake receiver in charge of countering multipath effects.

Trials in different configurations allowed to ascertain the impact of both chaos-based spreading and rake optimization.

Many more scenarios surely deserve attention before full applicability of chaos-based signal processing techniques can be claimed. Yet, in all tested cases, chaos-based spreading and related methodologies offered nonnegligible improvement to any combination of classical solutions.

**REFERENCES**


Gianluca Mazzini was born in 1968. He received the Dr. Eng degree in electronic engineering (with honors) and the Ph.D. degree in electronic engineering and computer science, both from the University of Bologna, Bologna, Italy, in 1992 and 1996, respectively.

Since 1996, he is with the University of Ferrara where he is an Assistant Professor of Telecommunications Networks and Electrical Communications. His research interests are related to spread-spectrum communications, chaos-based communication systems, internet mobile computing, cellular systems and protocols for wireless local area networks. He is author or co-author of over 90 technical papers.

Riccardo Rovatti was born in 1969. He received the Dr. Eng. degree in electronic engineering (with honors) and the Ph.D. degree in electronic engineering and computer science, both from the University of Bologna, Bologna, Italy, in 1992 and 1996, respectively.

Since 1997, he has been a Lecturer of Digital Electronics at the University of Bologna, and from 2000, is an Assistant Professor at the same university. His research interests include fuzzy-theory foundations, learning and CAD algorithms for fuzzy and neural systems, statistical-pattern recognition, function approximation, nonlinear system theory, and identification as well as theory and applications of chaotic systems. He has authored or co-authored more than 100 international scientific publications and is co-editor of the book *Chaotic Electronics in Telecommunications* (CRC: Boca Raton, FL, 2000).

Gianluca Setti received the Dr. Eng. degree (with honors) in electronic engineering and a Ph.D. degree in electronic engineering and computer science, both from the University of Bologna, Bologna, Italy, in 1992 and 1997, respectively, for his contribution to the study of neural networks and chaotic systems.

From May 1994 to July 1995, he was a Visiting Research Assistant with the Circuits and Systems Group of the Swiss Federal Institute of Technology, Lausanne, Switzerland. Since 1997, he has been a Lecturer and since 1998, he is an Assistant Professor of Analog Electronics at the University of Ferrara, Ferrara, Italy. His research interests include nonlinear circuit theory, recurrent neural networks, and design and implementation of chaotic circuits and systems, as well as their applications to electronics and signal processing.

Dr. Setti received the 1998 Caianiello prize for the best Italian Ph.D. dissertation on Neural Networks. He is currently chair of the IEEE Technical Committee on Nonlinear Circuits and Systems and is also serving as Associate Editor for the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—PART I, for the area of Nonlinear Circuits and Systems. He served as Technical co-chairman for the 2000 IEEE Specialist Workshop on Nonlinear Dynamics of Electronics Systems (NDES2000). He is also co-editor of the book *Chaotic Electronics in Telecommunications* (CRC: Boca Raton, FL, 2000).