Breaking Elliptic Curves Cryptosystems using Reconfigurable Hardware

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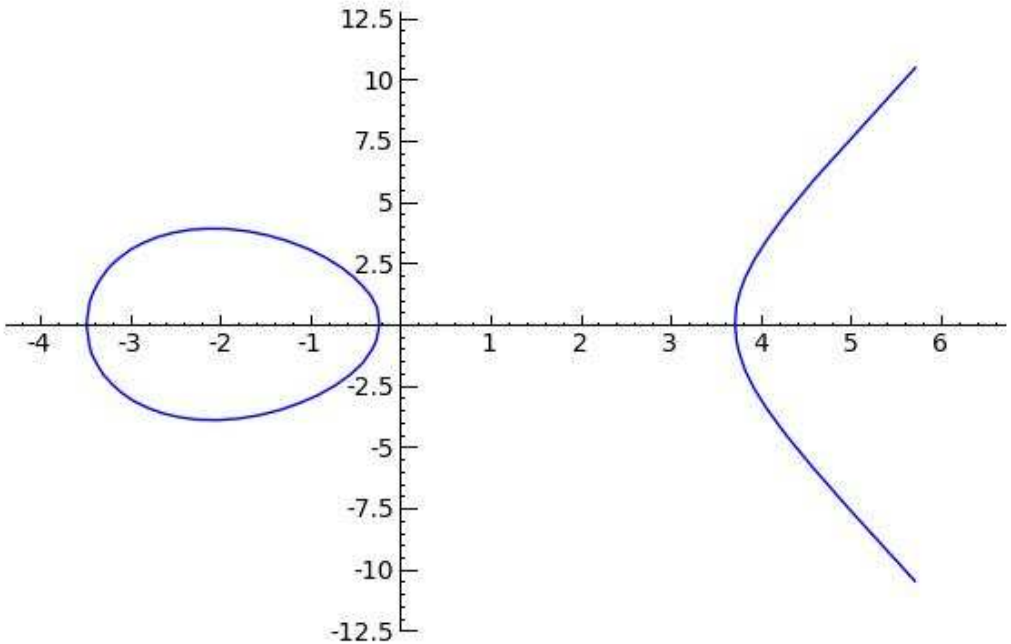
Elliptic Curve Cryptography



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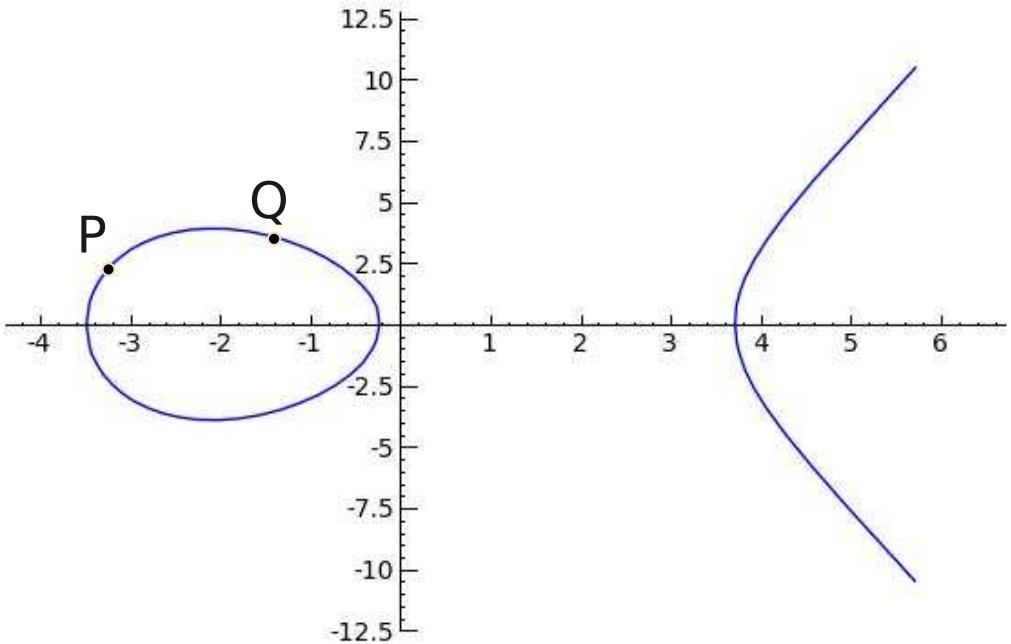

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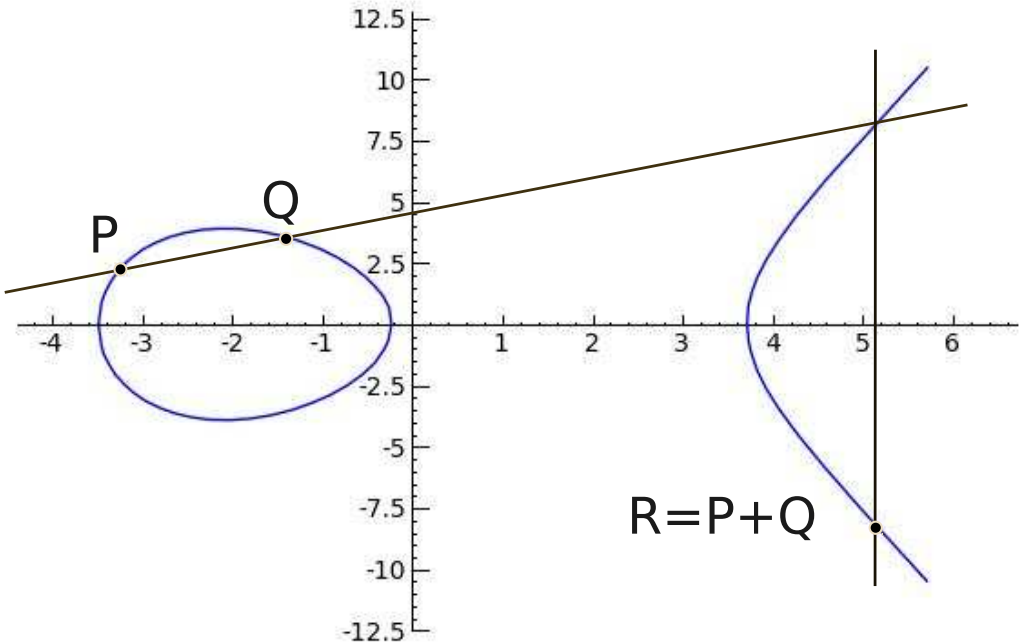

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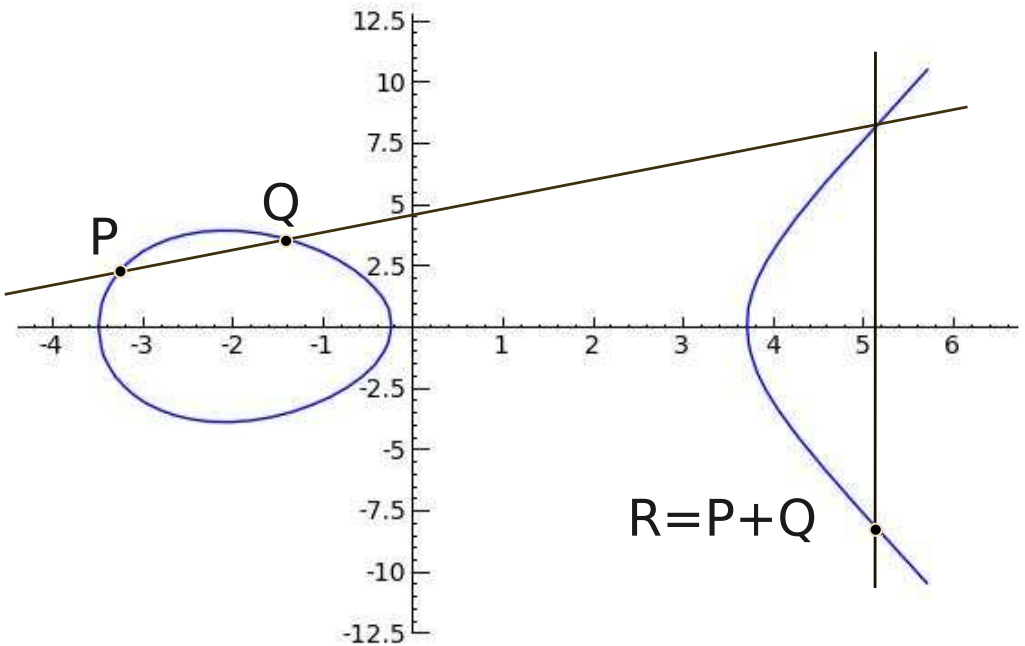

Point Multiplication:

$$Q \leftarrow [k]P$$
$$= P + P + \cdots + P.$$

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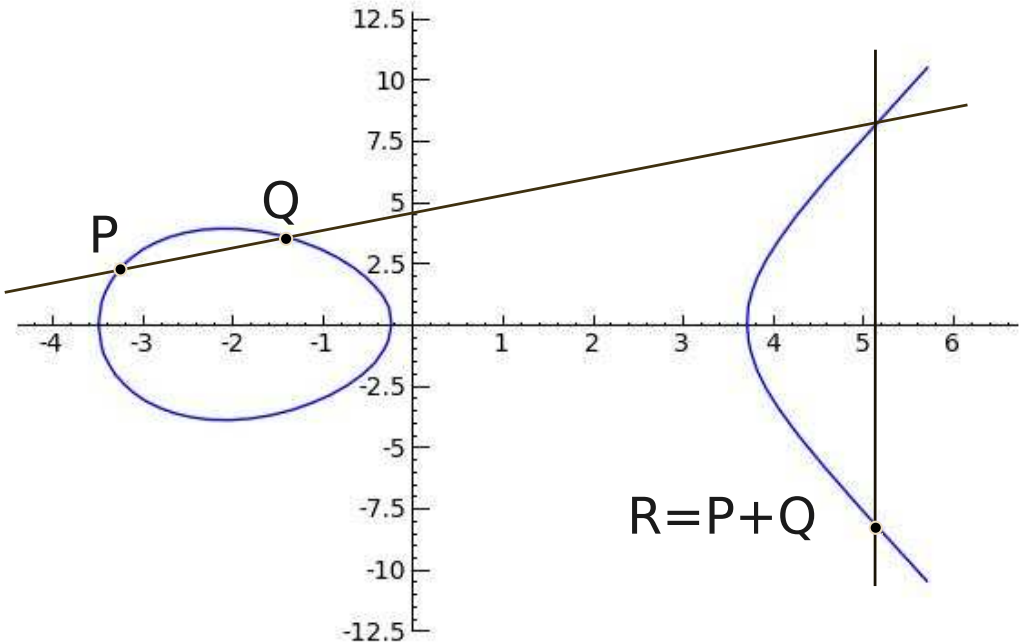

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Elliptic Curve Discrete Logarithm Problem (ECDLP):

Given Q and P, find k such that

$Q = [k]P$.

Certicom challenge

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ECC on GF(2^m)	ECC on GF(p)
ECC2-79	ECCp-79
ECC2-89	ECCp-89
ECC2K-95	ECCp-97
ECC2-97	ECCp-109
ECC2K-108	ECCp-131
ECC2-109	ECCp-163
ECC2K-130	ECCp-191
ECC2-131	ECCp-239
ECC2-163	ECCp-359
ECC2K-163	
...	
ECC2K-358	

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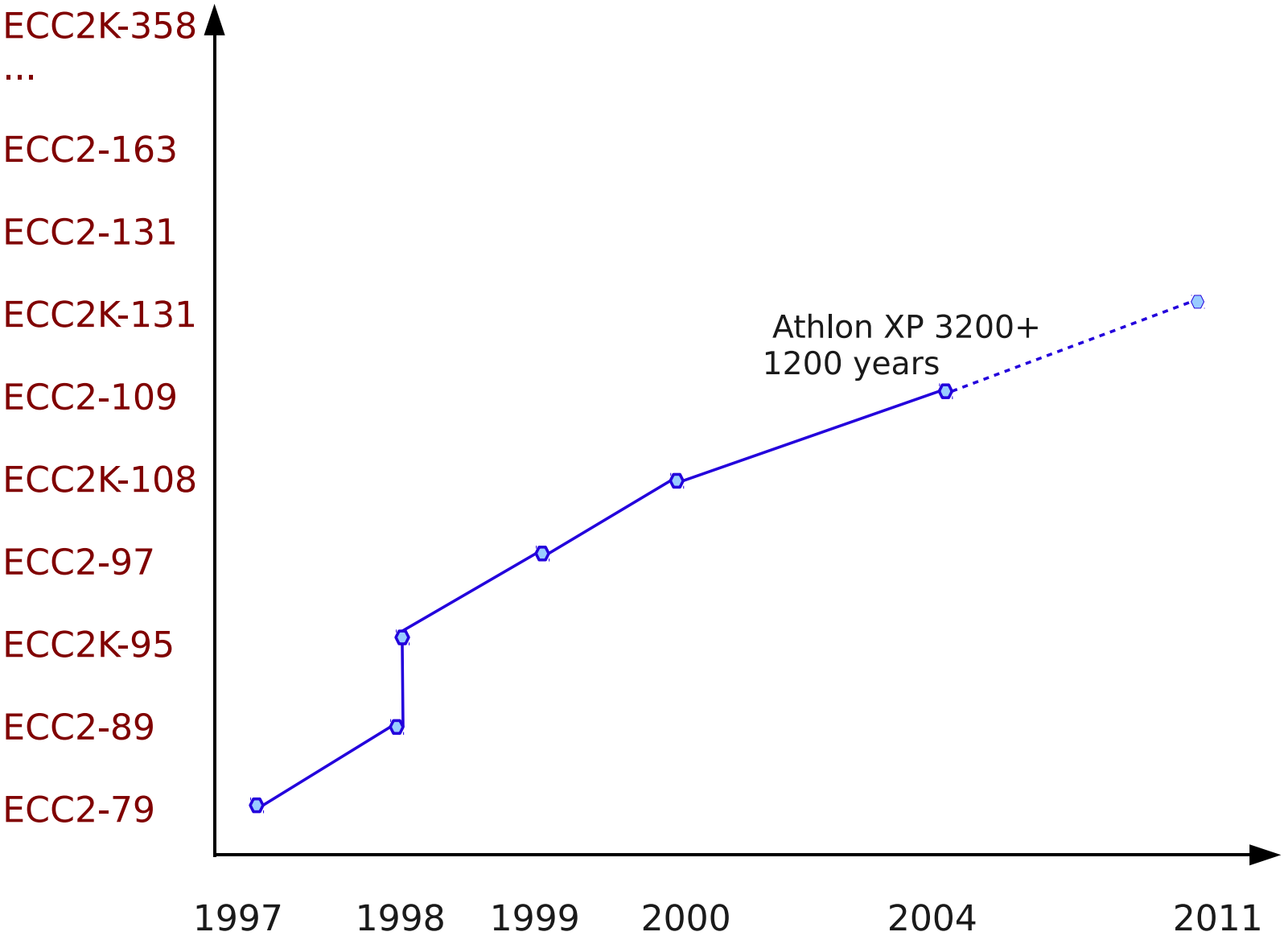

ECC2K-130

- $E : y^2 + xy = x^3 + 1$
- $F_{2^{131}} : f(w) = w^{131} + w^{13} + w^2 + w + 1$

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- **The challenge** : find k such that Q = [k]P.

The Ev1l Project

- Target : solve ECC2K-130
- Methods: Parallelized Pollard rho
- Platforms:
  - General Purpose CPU (Compute clusters)
  - Graphics card clusters
  - Playstation clusters
  - FPGA clusters
  - ASICs (Estimation)
- Status:
  - Globally distributed (Netherlands, Belgium, Ireland, USA, Taiwan, Switzerland, ..)
  - Progress report: http://www.ecc-challenge.info/

Parallelized Pollard rho

Pollard rho

finding (c_i, d_i) and (c_j, d_j) such that
$[c_i]P + [d_i]Q = [c_j]P + [d_j]Q$
$[(c_i - c_j)]P = [(d_j - d_i)]Q$
since $Q \equiv [k]P$,
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Random walk

- Start with $R_0 = [c_0]P + [d_0]Q$
- Iteration function
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  such that $R_i = [c_i]P + [d_i]Q$

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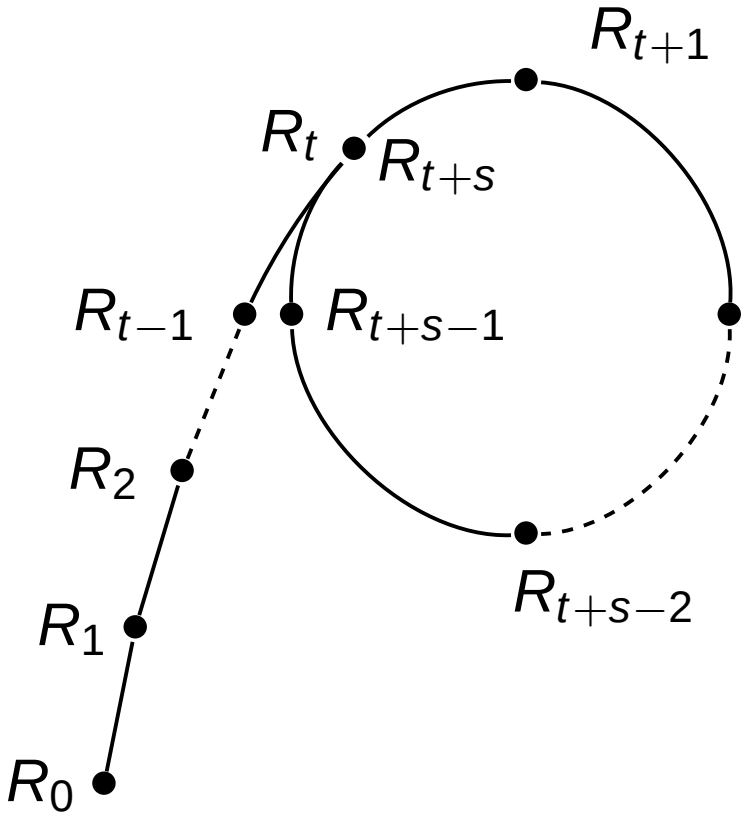

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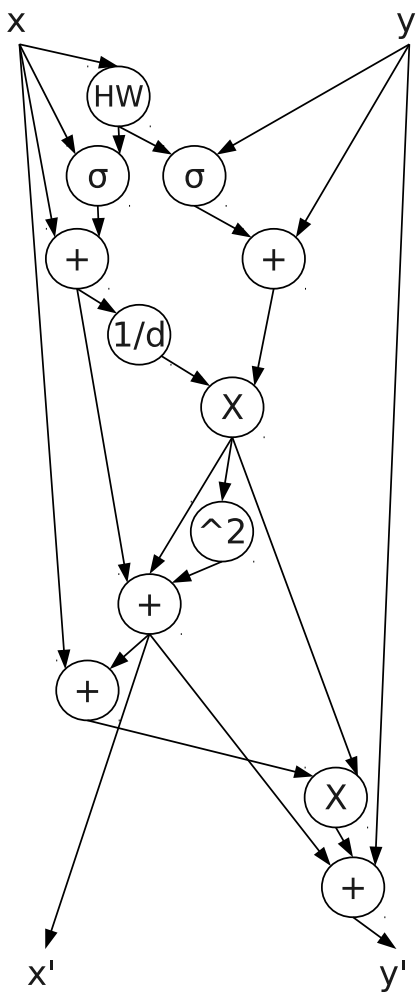

$j=(HW(x)/2 \bmod 8) +3$

$\sigma^j(x)$, $\sigma^j(y)$

$dx=x+\sigma^j(x)$,
$dy=y+\sigma^j(y)$

$s=1/dx$

$\lambda=s*dy$

λ^2

$x'=dx+\lambda+\lambda^2$

$x'+x$

$(x'+x)*\lambda$

$y'=y+x'+(x'+x)*\lambda$

FPGA platform : COPACOBANA-3

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- Xilinx Spartan-3 FPGA (XC3S5000)
- 128 FPGA units in total (16 x 8)
- 33,280 slices and 104 BRAMs available on one FPGA

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    P $=\{ 1, w, w^2, \cdots, w^{130}\}$.
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    N $=\{ \gamma + \gamma^{-1}, \gamma^2 + \gamma^{-2}, \gamma^{2^2} + \gamma^{-2^2}, \cdots, \gamma^{2^{130}} + \gamma^{-2^{130}}\}$.

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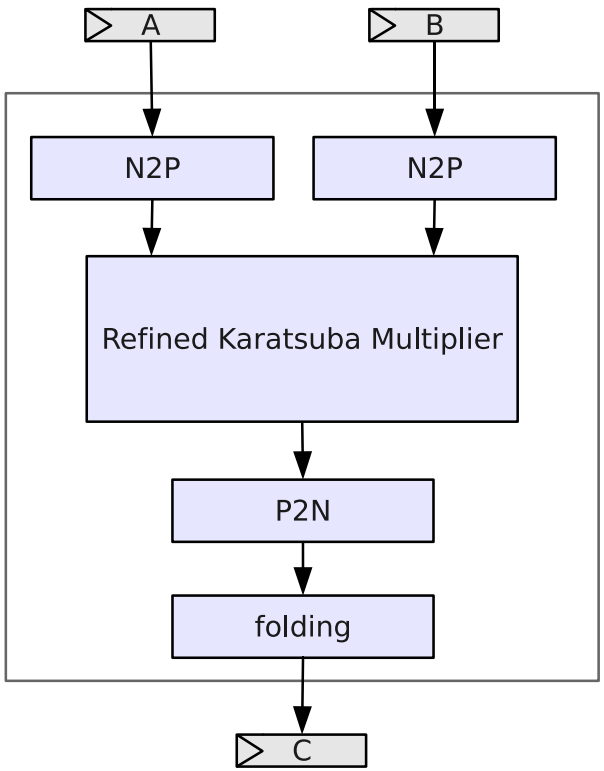
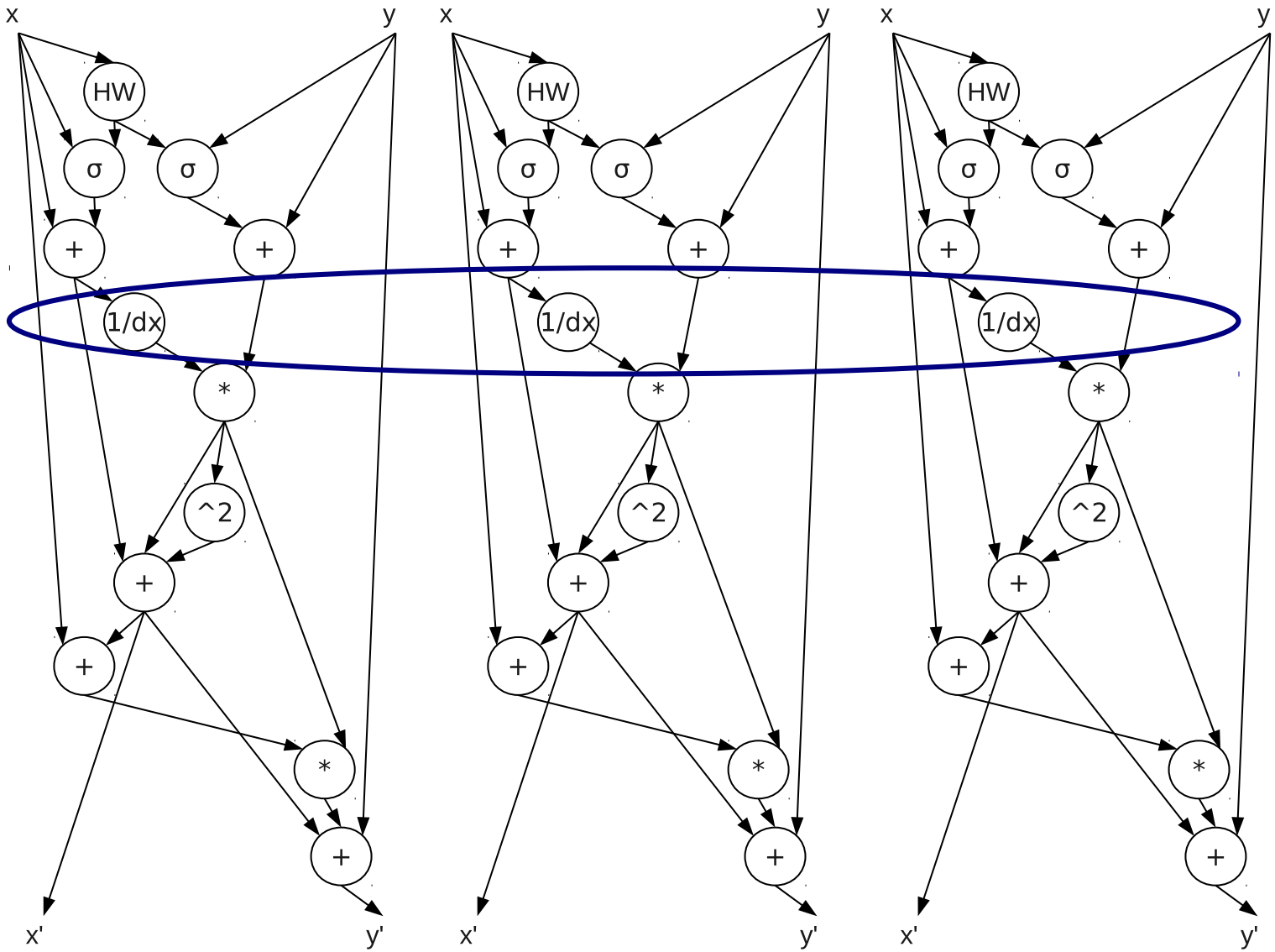
- Architecture
  - digit-serial
  - pipe-lined

Shokrollahi's multiplication algorithm

Fast conversion between

- permuted Normal **pN** basis
  $\{\gamma + \gamma^{-1}, \gamma^2 + \gamma^{-2}, \gamma^3 + \gamma^{-3}, \cdots, \gamma^{131} + \gamma^{-131}\}$.
- new Polynomial **nP** basis
  $\{(\gamma + \gamma^{-1}), (\gamma + \gamma^{-1})^2, \cdots, (\gamma + \gamma^{-1})^{131}\}$.




x
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Batch Inversion

Montgomery's trick for batch inversion (Batch size = 3)

Input: $\alpha_1, \alpha_2, \alpha_3$.
Output: α_1^{-1}, α_2^{-1} and α_3^{-1}.

1: $d_1 \leftarrow \alpha_1$
2: $d_2 \leftarrow d_1\alpha_2$
3: $d_3 \leftarrow d_2\alpha_3$
4: $u \leftarrow d_3^{-1}$
5: $t_3 \leftarrow ud_2$, $u \leftarrow u\alpha_3$
6: $t_2 \leftarrow ud_1$, $u \leftarrow u\alpha_2$
7: $t_1 \leftarrow u$

Return: t_1, t_2, t_3.

- we trade **I** for **3M + I**/n.
- One iteration: **2M + 3M + I**/n.

Polynomial basis representation, digit-serial multiplier, load/store architecture.

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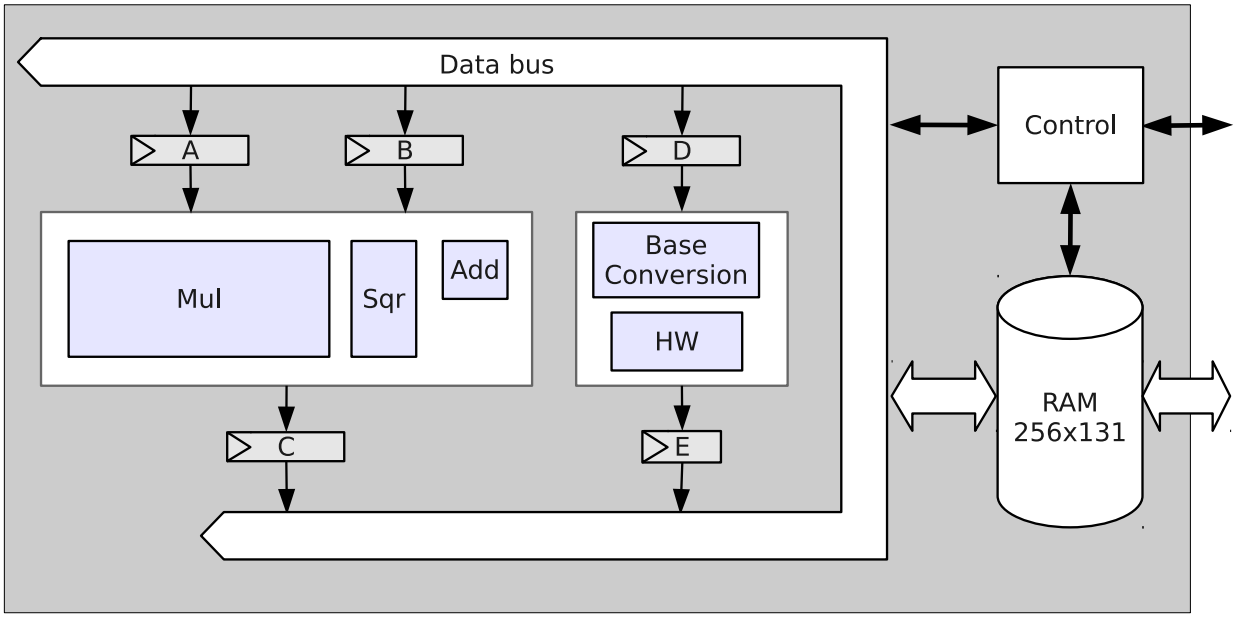

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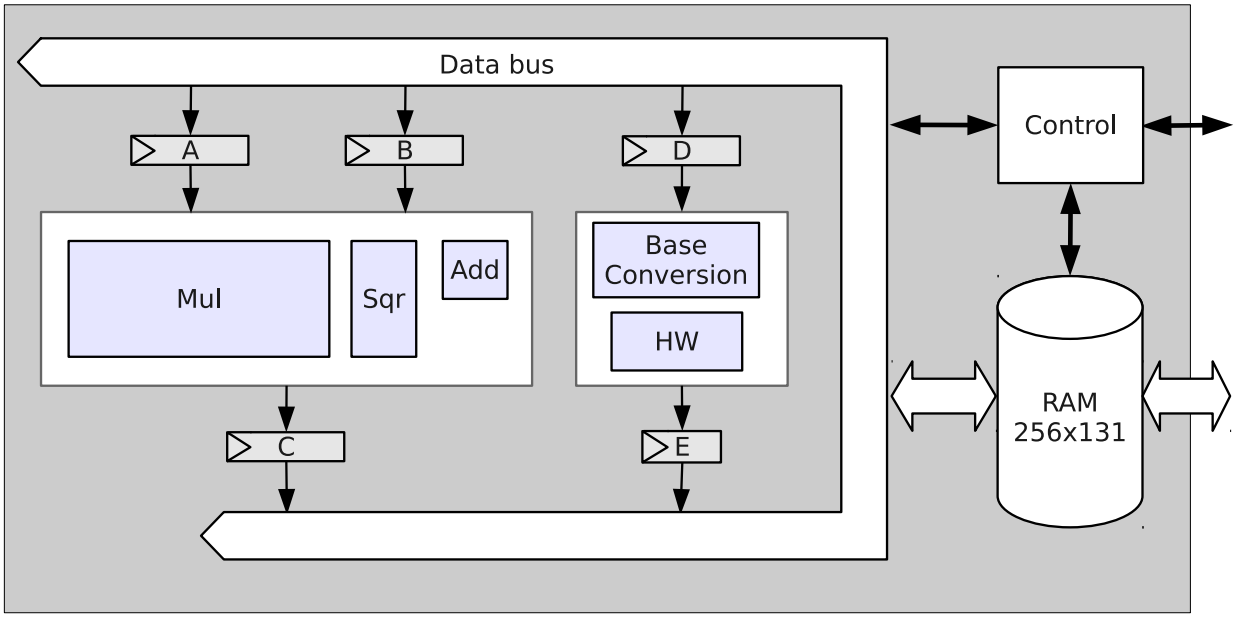

- Area : 3656 slices (Multiplier: 1468; conversion :1206)
- Frequency : 101 MHz
- 71 cycles per iteration

Normal basis representation, digit-serial multiplier (Kwon), load/store architecture

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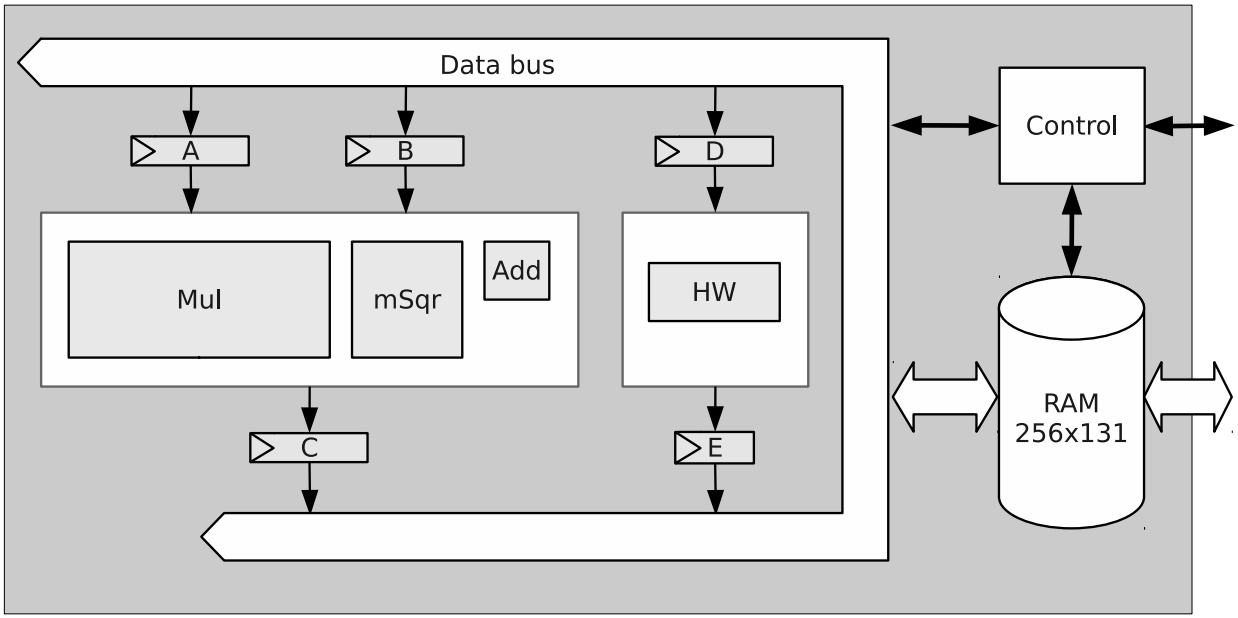

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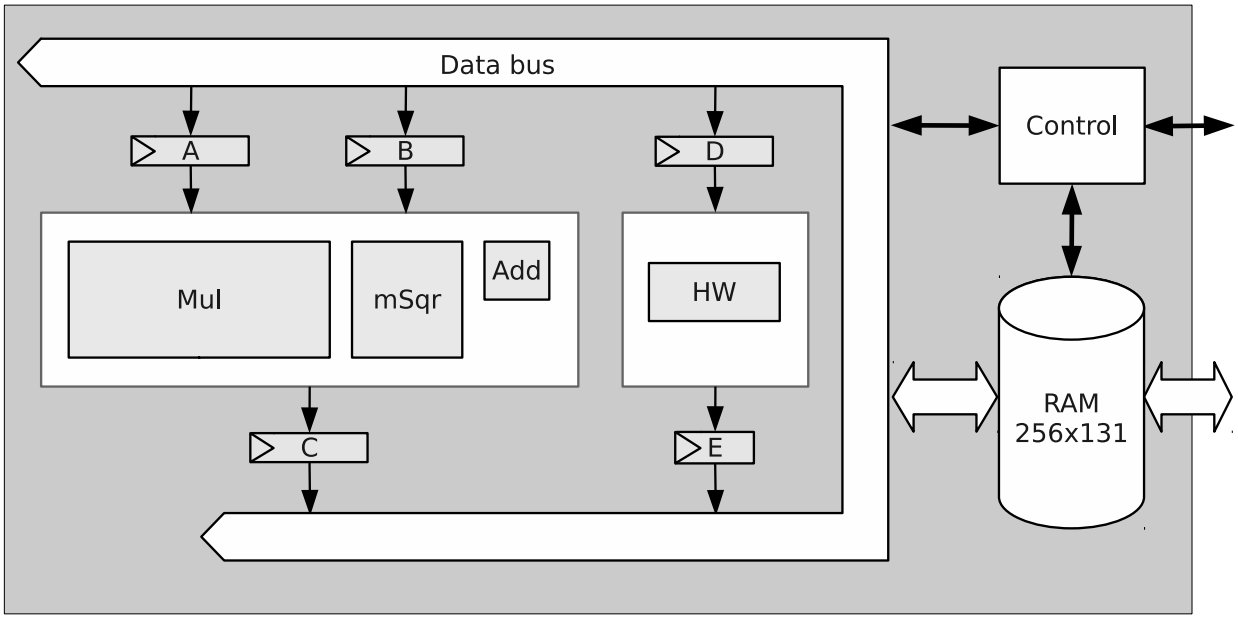

- Area : 2578 slices (Multiplier: 2093)
- Frequency : 125 MHz
- 81 cycles per iteration

Normal basis representation, Shorollahi multiplier (pipe-lined), Fully pipe-lined architecture

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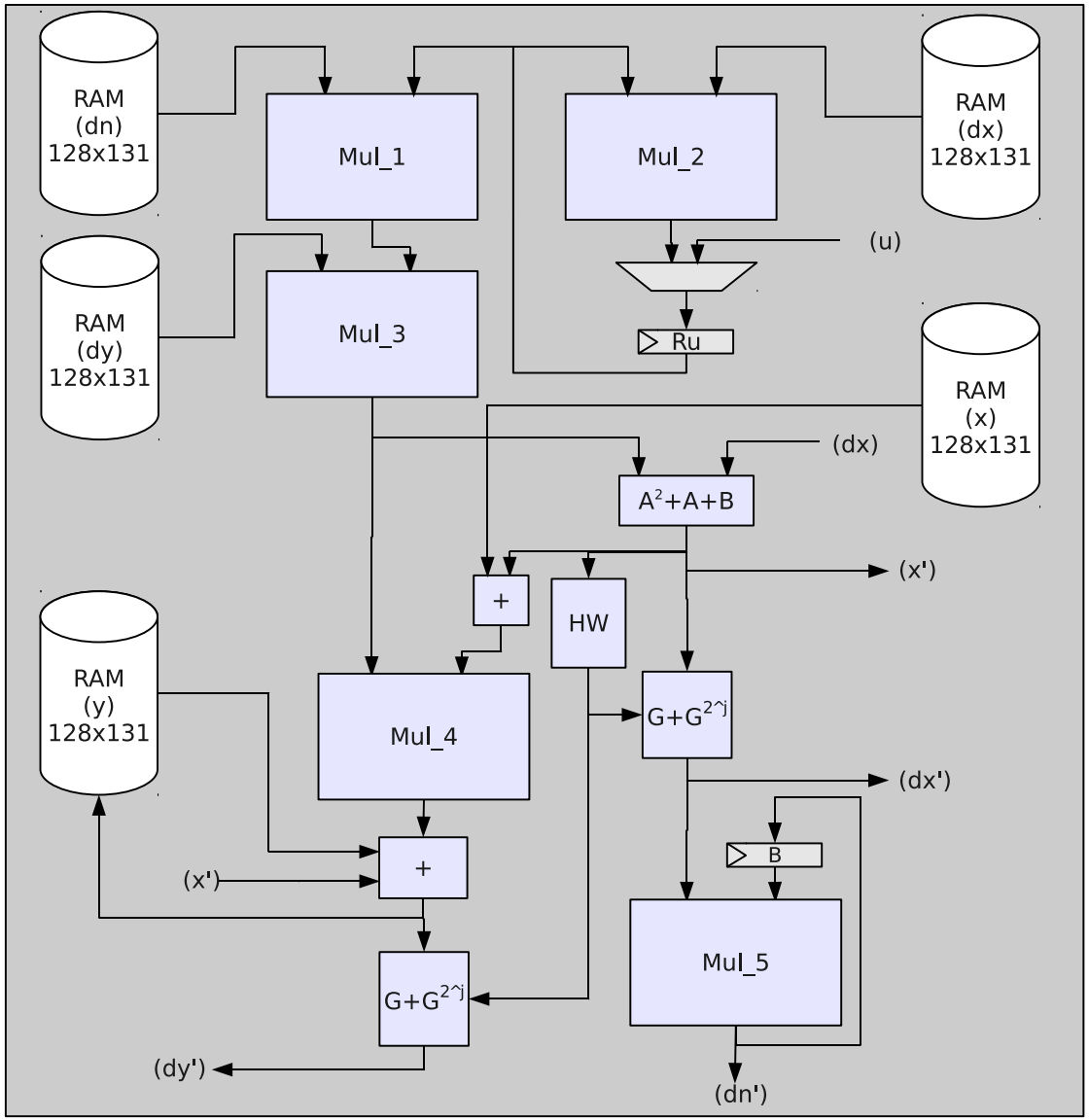
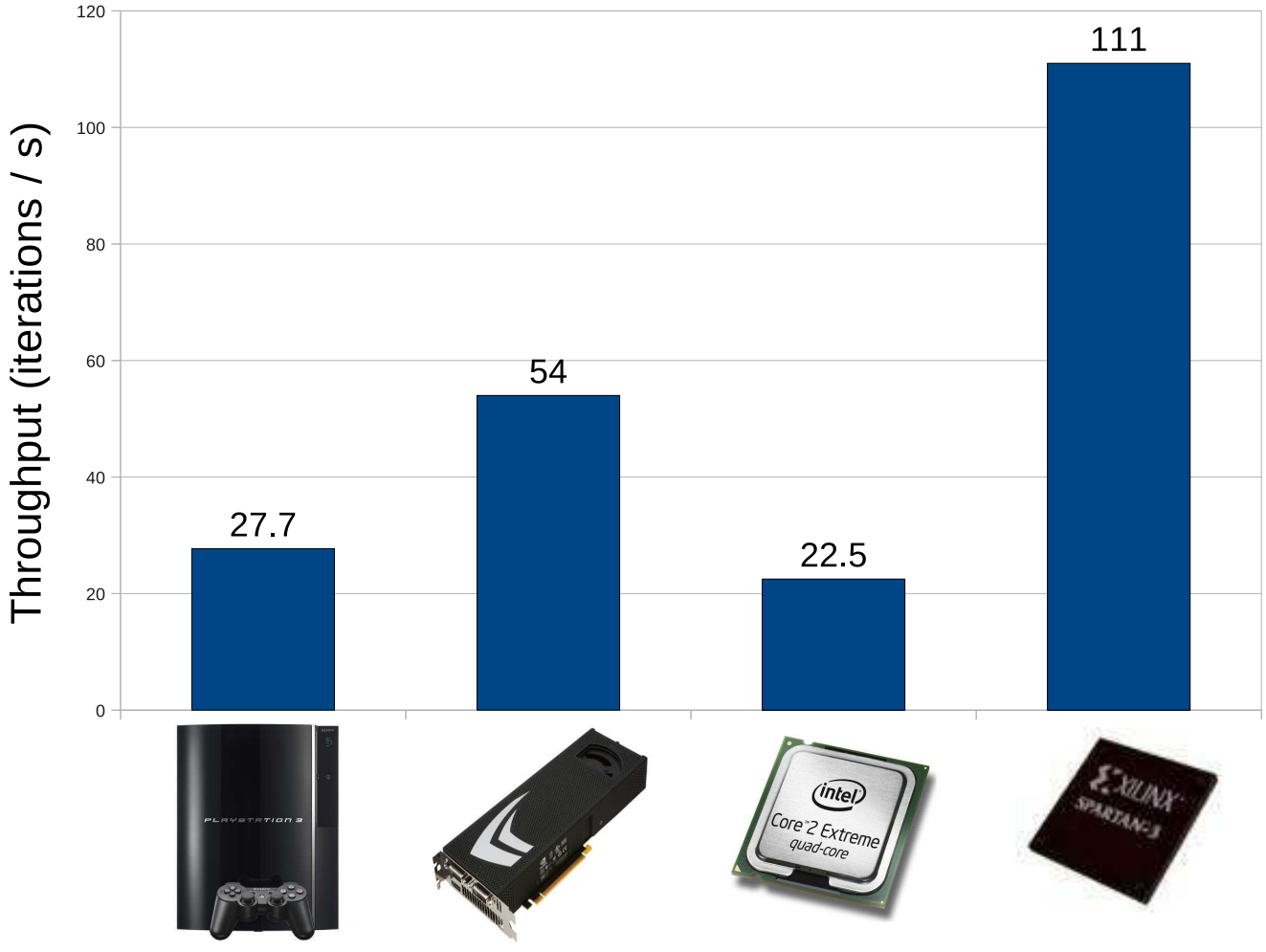


Performance of Archi-I, Archi-II and Archi-III

	Area	Freq.	Delay	Clones	Throughput
	#slice /#BRAM	[MHz]	[Cycles]		per FPGA
Archi-I: Polynomial	3,656 / 4	101	71	9	12.8×10^6
Archi-II: Type-II ONB	2,578 / 4	125	81	12	18.5×10^6
Archi-III: Shokrollahi's	26,731 / 20	111	23 *	1	111×10^6

* The average throughput is one iteration per cycle.

Comparison with CPU, Play station 3 and GPU



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  $\Rightarrow$ 128 x 111 x 2^{20} x 3600 x 24 x 365 = $2^{58.7}$ per year
- $2^{60.9}$ iterations are expected
  $\Rightarrow$ Given 5 COPACOBANA-3 clusters, we can solve the ECC2K-130 in one year!

Conclusion

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- Low-cost FPGA show good performance/cost ratio compared with CPU, Play Station 3 and GPU.
- ECC2K-130 is practically solvable.

D. V. Bailey, L. Batina, D. J. Bernstein, P. Birkner, J. W. Bos, H. Chen, C. Cheng, G. van Damme, G. de Meulenaer, L. J. D. Perez, J. Fan, T. Güneysu, F. Gurkaynak, T. Kleinjung, T. Lange, N. Mentens, R. Niederhagen, C. Paar, F. Regazzoni, P. Schwabe, L. Uhsadel, A. Van Herrewege, and B. Yang.

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