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Mapping Multiple Multivariate Gaussian Random Number Generators on an FPGA

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Outline

- Monte Carlo Simulation
- Multivariate Gaussian Random Number Generator (MVGRNG)
- Objective
- Optimization algorithm
- Proposed framework – Hardware architecture
- Experimental Results
- Conclusions

Introduction

- Monte Carlo simulation
  - » Mathematical technique
  - » Repeated random sampling
  - » Evaluate non-deterministic processes
- Pre-requisite for MC simulation → **random numbers**
- Multivariate Gaussian distribution to capture many *correlated* variables
- Acceleration of MC using FPGA
  - » Speed up simulations
  - » Optimization of MVGRNG

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Objective

- Existing approaches only focus on single distribution MVGRNG
- Mapping of *multiple* multivariate Gaussian distributions
- Example: Optimization of many financial portfolios
  - » Represented by many multivariate Gaussian distributions
- MVRNG usually part of larger application
  - » Resource usage → CRUCIAL
- Efficient resource sharing

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Generate multivariate Gaussian Random Numbers

- Mean ($\mathbf{m}$) and Covariance matrix ($\boldsymbol{\Sigma}$)
- **OBJECTIVE : APPROXIMATE Σ**
- Eigenvalue Decomposition using SVD

$$\Sigma = \mathbf{U} \cdot \Lambda \cdot \mathbf{U}^T$$

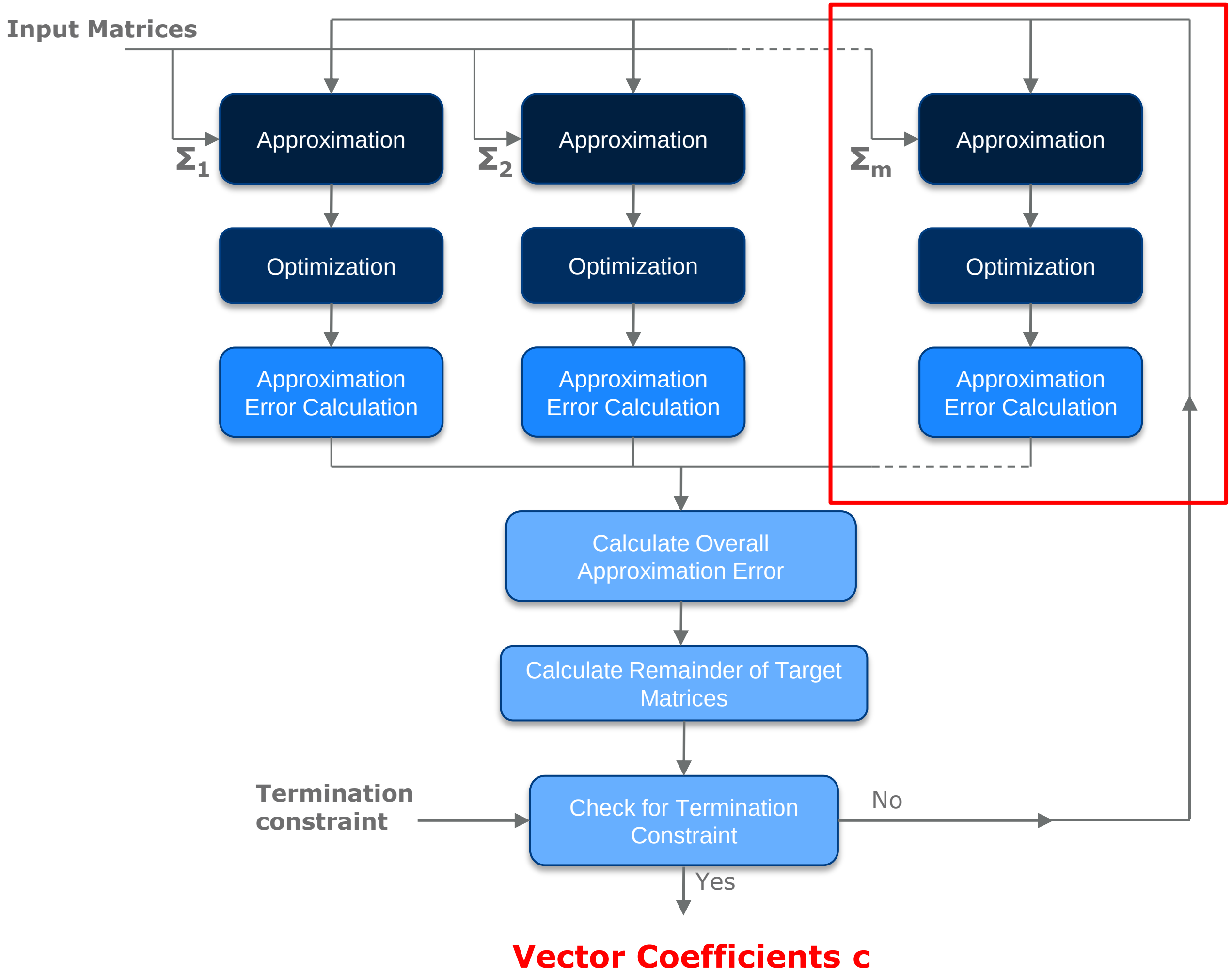
$$= \mathbf{U}\Lambda^{1/2}\Lambda^{1/2}\mathbf{U}^T$$

$$\mathbf{x} = \mathbf{U}\Lambda^{1/2}\mathbf{z} + \mathbf{m}, \quad \mathbf{z} \sim N(0,\mathbf{I})$$

$$= \sum_{i=1}^{K} (\mathbf{c}_i z_i) + \mathbf{m}$$

- Using any levels of decomposition K

Proposed Algorithm



Algorithm takes any M number of distributions

Approximate **Σ** for each distribution

Target redundancies between *ALL* input distribution

Exploit similarities in **PRECISION REQUIREMENTS**

Select appropriate precision to *minimize approximation error for all distributions*

Distinct coefficients for each distribution

Error Estimation Model

- Mean square error
- Approximation error for each distribution

Actual Matrix

$$Error = \frac{1}{N^2} \left\| \Sigma - \bar{\Sigma} \right\|^2$$

Matrix Order

Approximated Matrix

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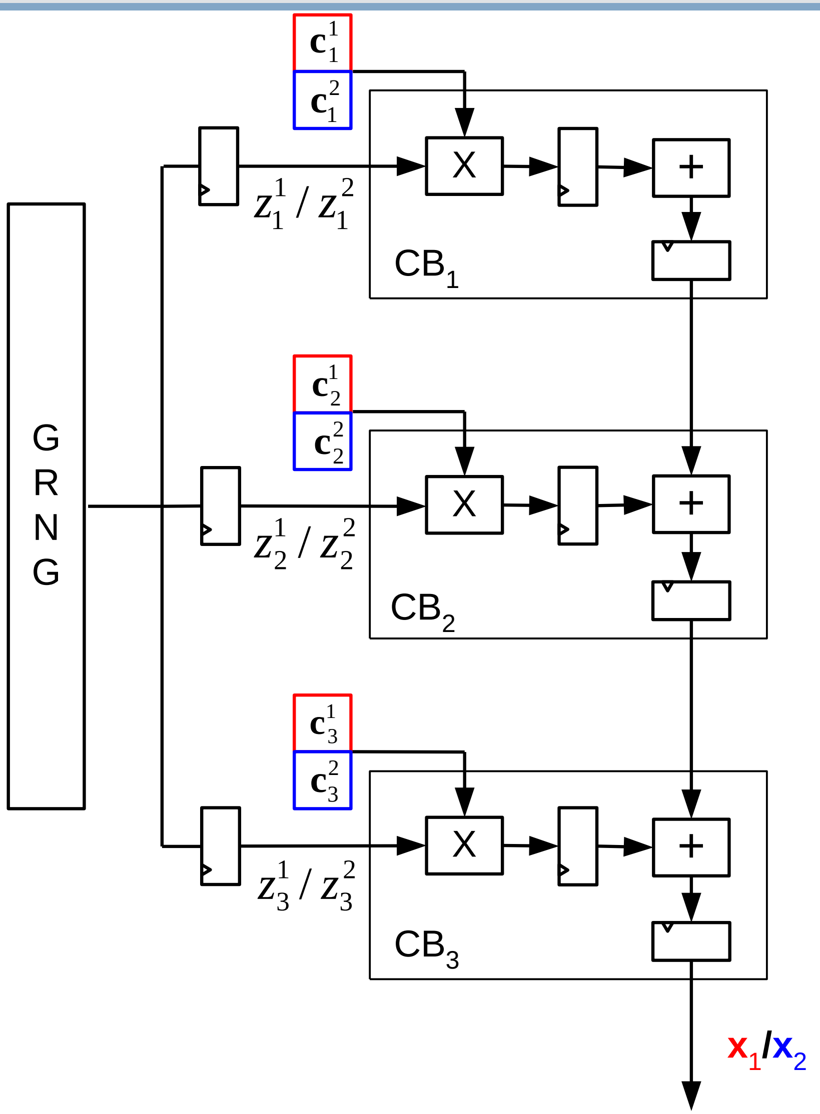
Hardware Architecture

Constructed from *K* number of CBs
K = no of decomposition levels

Mixed precisions in datapath

LUTs based

Precision in adder path = max(All CBs)



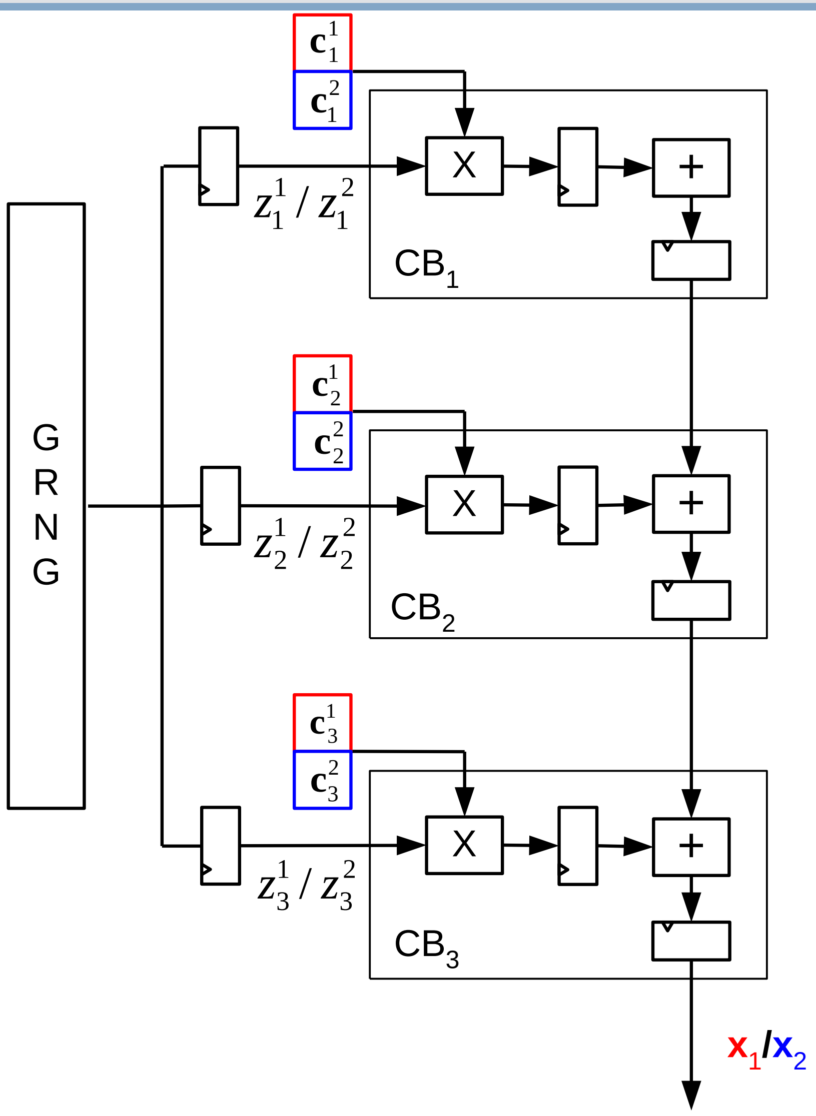
Hardware Architecture

Two multivariate Gaussian distributions (3x3 correlation matrices)

Using 3 levels of decomposition (K=3)

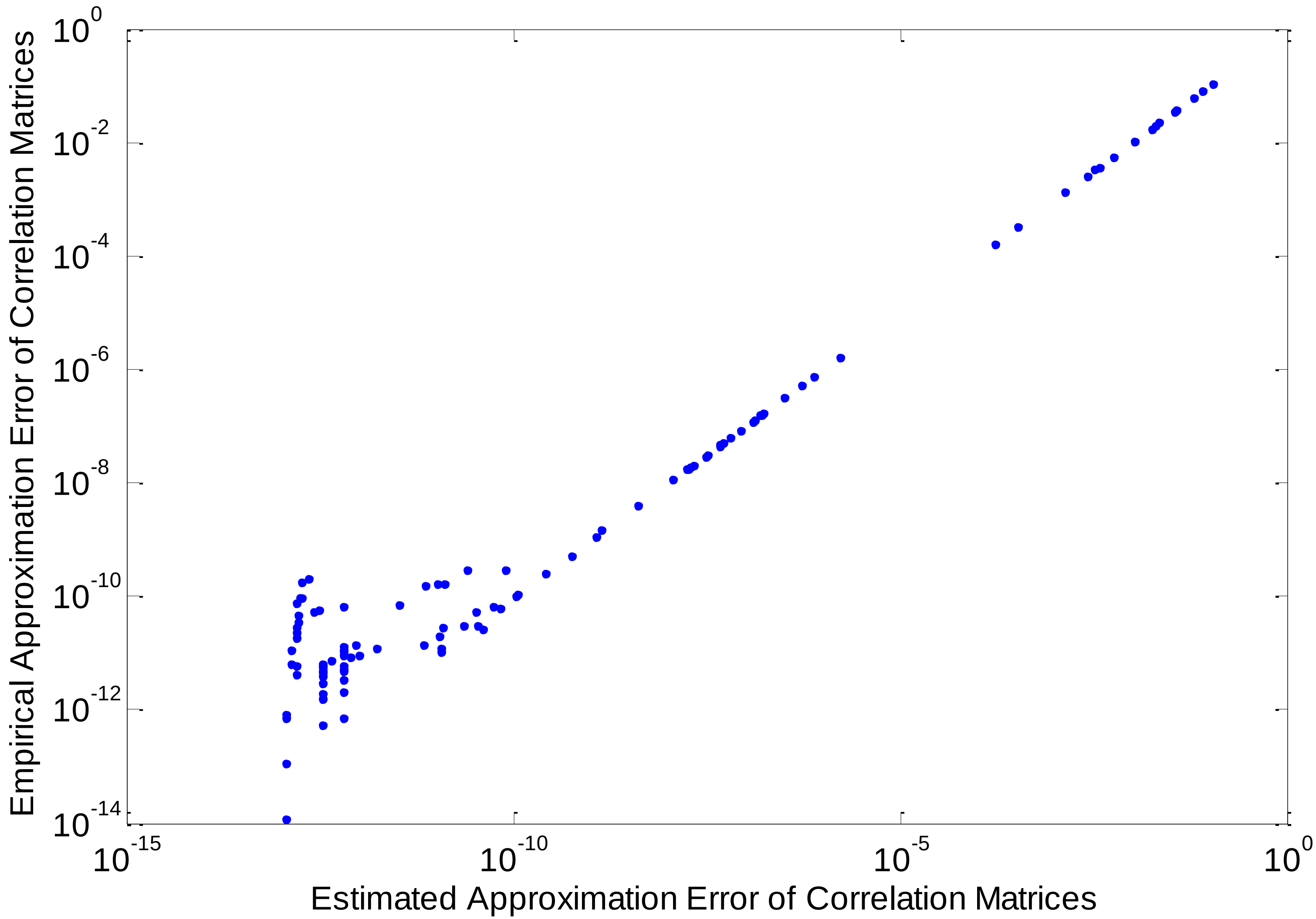
GRNG with different seeds for each input distribution – ***completely independent***

$\mathbf{x}_1$ produced after K clock cycles
$\mathbf{x}_2$ produced after 2K clock cycles



Experiment I
Accuracy of Error and Resource Estimation Model

Accuracy of the Error Estimation Model



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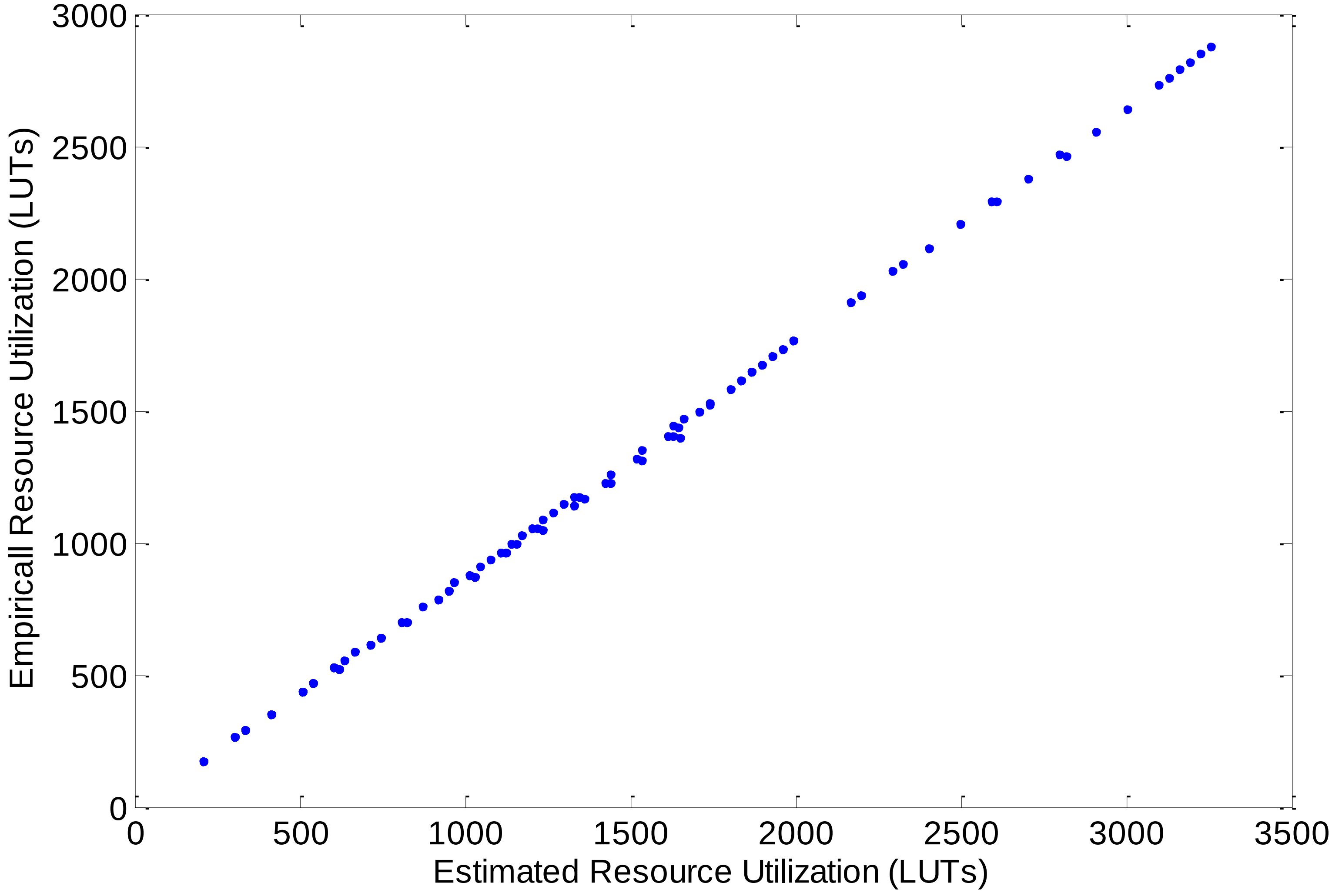
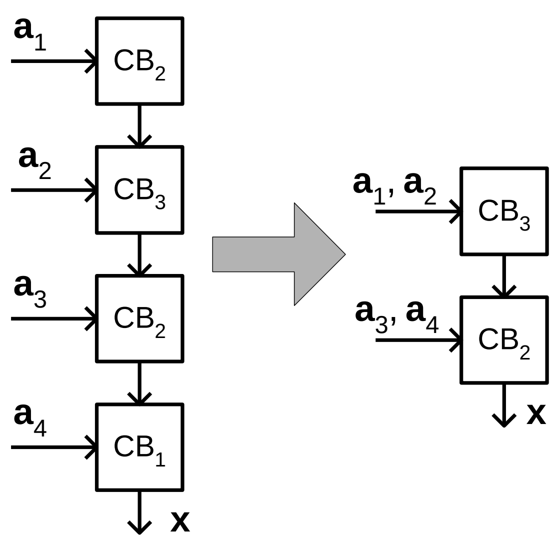
Accuracy of the Resource Estimation Model



Experiment II
Comparison with Existing Approaches

Experimental Setup

- Approaches under consideration
  - » [Thomas and Luk 2008]
  - » Our previous work [Saiprasert et al 2009]
- Adjust throughput of existing approaches to be the same level
  - » Fair comparison
- Force M consecutive levels to use same CB for [Saiprasert et al 2009]
  - » M = number of input distributions



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Comparison of All Approaches

	[Thomas08]	[Saiprasert09]	This work
Architecture	DSP	LUTs	LUTs
Precision	Fixed	Mixed	Mixed
Optimization across all input distributions	No	No	Yes
	Reuse same hardware for all input matrices	Force M consecutive decomposition levels to share same hardware	Optimized precisions and coeff for all input distributions

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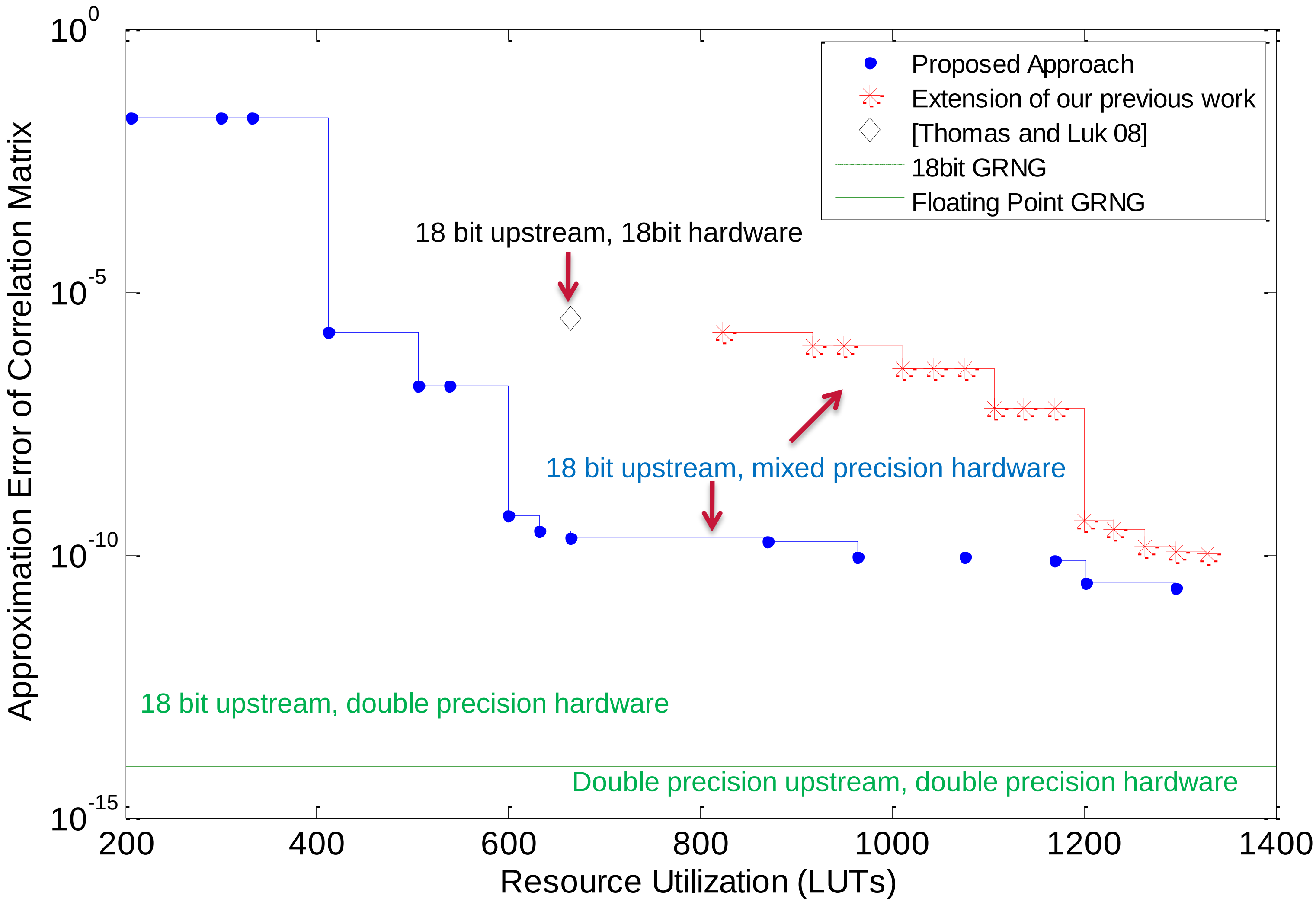
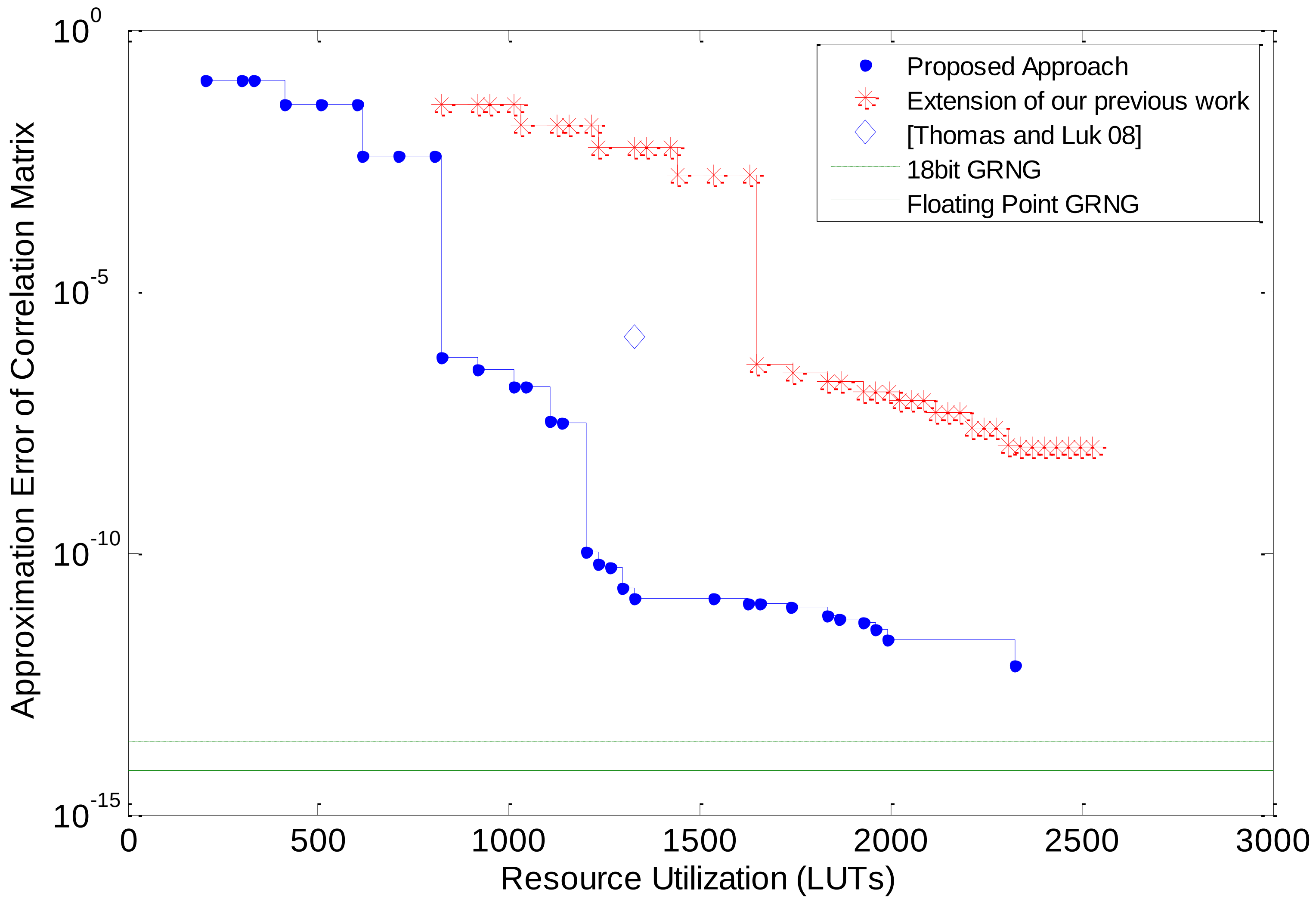
Experimental Setup

- 4 sets of input correlation matrices
  - » Set I: Four 2x2 matrices
  - » Set II: Four 4x4 matrices
  - » Set III: Four 6x6 matrices
  - » Set IV: Two 2x2 and two 4x4 matrices
- One MVGRNG optimised for each set
- 100,000 vectors obtained for each set

Set I Matrices (2x2)



Set II Matrices (4x4)



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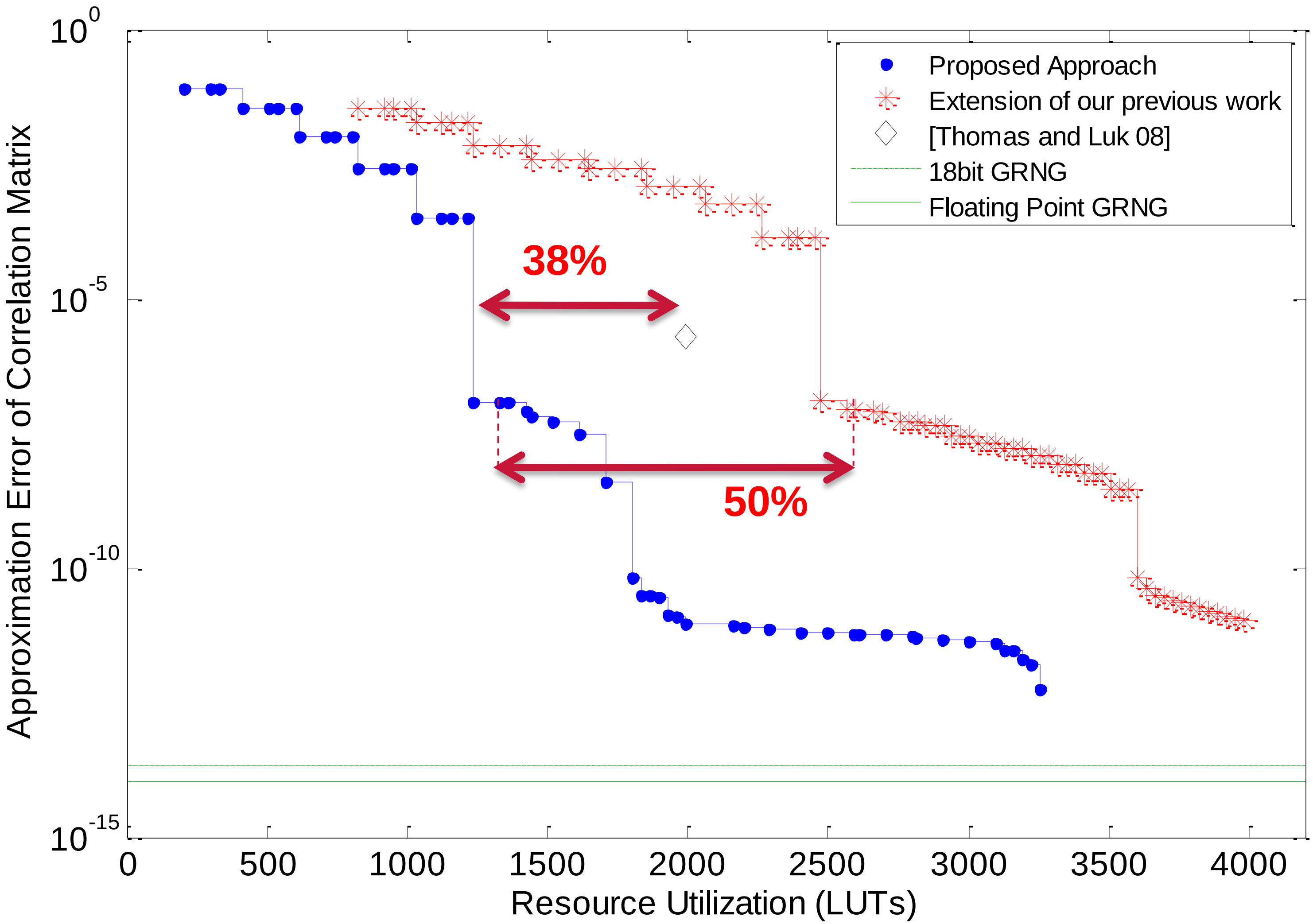
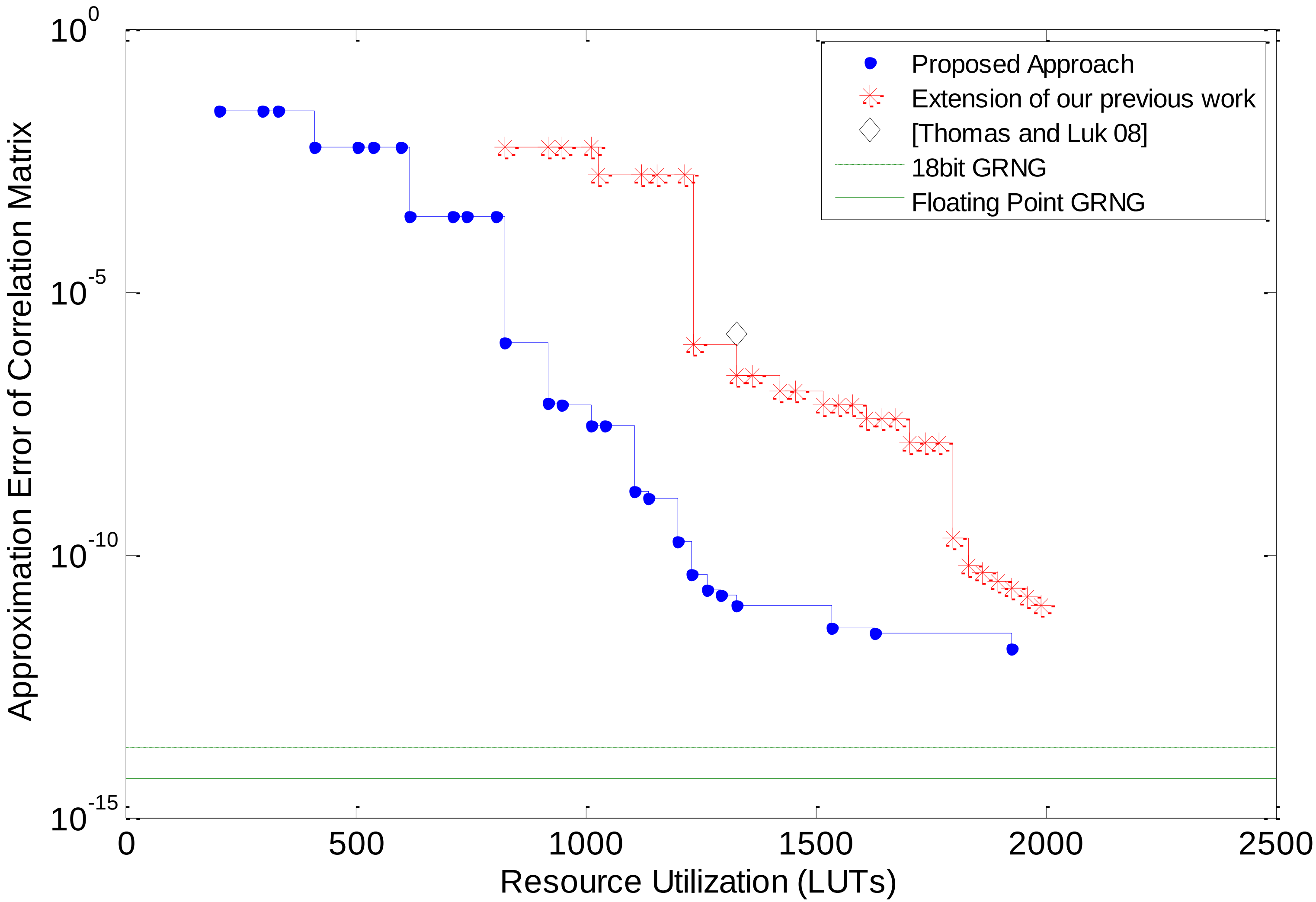
Set III Matrices (6x6)



Set IV Matrices (Mixed Matrix Orders)



Conclusions

- Innovative approach for multiple distributions MVGRNG
- One generator optimized for all input distributions
- Effective resource sharing algorithm
- Exploits similarities in precision requirements
- Up to 50% reduction in resource usage
- Without any penalty on the quality of the generated data

THANK YOU FOR YOUR ATTENTION