Algebraic Fault Analysis on GOST for Key Recovery and Reverse Engineering

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Outline

- **Motivation?** Algebraic Fault Analysis
- **Target?** GOST and Attack Scenarios
- **Technique?** AFA on GOST
- **Results?** Key Recovery and Reverse Engineering
- **Summary?** Conclusion of Our Work
Traditional Fault Analysis

FA (Fault Attack) first proposed by Boneh et al in 1996.
  – Received faulty output, guess the fault, find the secret.

• DFA (Differential Fault Analysis) proposed by Biham and Shamir in 1997.
  – Used to break public-key ciphers (ECC), block ciphers (AES, ARIA, Camellia and CLEFIA) and stream ciphers (RC4, Trivium).

Framework of DFA

Manually fault analysis; Maximal efficiency unknown?
Algebraic Fault Analysis

• AFA (Algebraic Fault Analysis) proposed by Courtois in 2010.
  – Algebraic cryptanalysis with fault attack.

Compared with DFA:

➤ Algebraic analysis are **generic and automatic**
➤ Solvers (automatic) allow **easier and simpler analysis**
➤ Fault information allows optimization
State-of-the-art AFA

- **eSmart 2010**
  - Courtois: DES, single fault, $2^{17.35}$ hours

- **COSADE 2011**
  - Mohamed: Trivium, less faults

- **ePrint 2012/400**
  - Jovanovic: LED, single fault, 14.67 hours

- **COSADE 2013**
  - Zhang: Piccolo, DES (10 seconds), MIBS, single fault

- **FDTC 2013**
  - Zhao: LED, single fault, 1-3 minutes, evaluating DFA

- **CACR 2013**
  - Zhao: LBlock, single fault

- **AFA**
  - Fast
  - Lower data complexity
Our Motivations?

• Current AFA
  – Key recovery when the design of cipher is known
  – Evaluating the reduced key search space of DFA

• Our work
  – Can AFA work when partial design of cipher is unknown?
  – Can AFA be used for reverse engineering besides key recovery?
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Overview of GOST

- A Soviet and Russian government standard symmetric key block cipher.
  - 64-bit block cipher
  - 256 bit keys
  - 32 rounds
  - Feistel structure
  - 8 S-Boxes
  - modulo $2^{32}$ nonlinear part
  - Simple key schedule
Overview of GOST

• processes the right half of the block using function f, XORs the result from f with the left half, and swaps the two halves.
• key schedule is simple, divide 256-bit key into 8 pieces, using one piece per round

Figure 1. One round of GOST

the contents of 8 S-Boxes might be secret
Attack Scenarios

single byte fault injection on the right half of GOST

- **Scenario 1**: known complete GOST design, **key recovery**?

- **Scenario 2**: 8 S-Boxes secret, known secret key, AFA technique, **reverse engineering** of S-Boxes?

- **Scenario 3**: 8 S-Boxes secret, unknown secret key, AFA technique, **both key recovery and reverse engineering**?

Figure 1. One round of GOST
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AFA on GOST

- one full correct GOST equation set
- the last few GOST rounds equation set since the fault injections for N pairs of correct and faulty encryptions

Figure 2. Framework of AFA on GOST
Step 1: GOST Equation Set

- Represent AK (Adding modulo $2^{32}$)

\[
\begin{align*}
    z_{32} &= x_{32} + y_{32} \\
    t_{31} &= x_{32}y_{32} \\
    z_{31} &= x_{31} + y_{31} + t_{31} \\
    t_{30} &= x_{31}y_{31} + x_{31}t_{31} + y_{31}t_{31} \\
    z_{30} &= x_{30} + y_{30} + t_{30} \\
    t_{29} &= x_{30}y_{30} + x_{30}t_{30} + y_{30}t_{30} \\
    z_{29} &= x_{29} + y_{29} + t_{29} \\
    t_{28} &= x_{29}y_{29} + x_{29}t_{29} + y_{29}t_{29} \\
    \ldots \\
    z_{2} &= x_{2} + y_{2} + t_{2} \\
    t_{1} &= x_{2}y_{2} + x_{2}t_{2} + y_{2}t_{2} \\
    z_{1} &= x_{1} + y_{1} + t_{1}
\end{align*}
\]
Step 1: GOST Equation Set

- Represent SL (S-Box lookup)

\[
\begin{align*}
y_1 &= x_2 + x_3 + x_4 + x_1 x_2 + x_1 x_3 + x_2 x_4 + x_1 x_2 x_4 + x_2 x_3 x_4 \\
y_2 &= 1 + x_3 + x_4 + x_3 x_4 + x_1 x_2 x_3 + x_1 x_2 x_4 + x_1 x_3 x_4 + x_2 x_3 x_4 \\
y_3 &= x_1 + x_4 + x_1 x_3 + x_1 x_4 + x_2 x_4 + x_1 x_2 x_4 + x_2 x_3 x_4 \\
y_4 &= x_2 + x_3 + x_1 x_4 + x_2 x_4 + x_3 x_4 + x_1 x_2 x_3 + x_1 x_3 x_4
\end{align*}
\]

Public S-Box

\[
\begin{align*}
y_1 &= a_1 + a_2 x_1 + a_3 x_2 + a_4 x_3 + a_5 x_4 + a_6 x_1 x_2 + a_7 x_1 x_3 + a_8 x_1 x_4 + a_9 x_2 x_3 + a_{10} x_2 x_4 + a_{11} x_3 x_4 + a_{12} x_1 x_2 x_3 + a_{13} x_1 x_2 x_4 + a_{14} x_1 x_3 x_4 + a_{15} x_2 x_3 x_4 + a_{16} x_1 x_2 x_3 x_4 \\
y_2 &= a_{17} + a_{18} x_1 + a_{19} x_2 + a_{20} x_3 + a_{21} x_4 + a_{22} x_1 x_2 + a_{23} x_1 x_3 + a_{24} x_1 x_4 + a_{25} x_2 x_3 + a_{26} x_2 x_4 + a_{27} x_3 x_4 + a_{28} x_1 x_2 x_3 + a_{29} x_1 x_2 x_4 + a_{30} x_1 x_3 x_4 + a_{31} x_2 x_3 x_4 + a_{32} x_1 x_2 x_3 x_4 \\
y_3 &= a_{33} + a_{34} x_1 + a_{35} x_2 + a_{36} x_3 + a_{37} x_4 + a_{38} x_1 x_2 + a_{39} x_1 x_3 + a_{40} x_1 x_4 + a_{41} x_2 x_3 + a_{42} x_2 x_4 + a_{43} x_3 x_4 + a_{44} x_1 x_2 x_3 + a_{45} x_1 x_2 x_4 + a_{46} x_1 x_3 x_4 + a_{47} x_2 x_3 x_4 + a_{48} x_1 x_2 x_3 x_4 \\
y_4 &= a_{49} + a_{50} x_1 + a_{51} x_2 + a_{52} x_3 + a_{53} x_4 + a_{54} x_1 x_2 + a_{55} x_1 x_3 + a_{56} x_1 x_4 + a_{57} x_2 x_3 + a_{58} x_2 x_4 + a_{59} x_3 x_4 + a_{60} x_1 x_2 x_3 + a_{61} x_1 x_2 x_4 + a_{62} x_1 x_3 x_4 + a_{63} x_2 x_3 x_4 + a_{64} x_1 x_2 x_3 x_4
\end{align*}
\]

Secret S-Box

64 variables $a_i$ are introduced
Step 1: GOST Equation Set

- Represent RL (Rotating bits to left)

\[ y_i = x_{((i+9) \mod 32)+1} \]

- Represent GOST decryption can accelerate speed of AFA

Algorithm 1. Building the equation set for \( r \) rounds decryption of GOST

1: \( C \leftarrow [c_1, c_2, \ldots, c_{64}] \)
2: \( L_{33} \leftarrow [c_1, c_2, \ldots, c_{32}] \)
3: \( R_{33} \leftarrow [c_{33}, c_{34}, \ldots, c_{64}] \)
4: \( L_{32} \leftarrow L_{33} \oplus RL(SL(RL(R_{33}, K_{32}))) \)
5: \( R_{32} \leftarrow R_{33} \)
6: for \( i = 31 \) to \( 32 - r \) (\( i > 0 \)) do
7: \( L_i \leftarrow R_{i+1} \)
8: \( R_i \leftarrow L_{i+1} \oplus RL(SL(RL(R_{i+1}, K_i))) \)
9: end for
Step 2: Fault Equation Set

- Suppose $Z$ denote the injected fault difference
  
  - $Z$ can be considered as the concatenation of four bytes
    
    $$Z_1 || Z_2 || Z_3 || Z_4, \quad Z_i = (z_{8i-7}, z_{8i-6}, \ldots, z_{8i}) \ (1 \leq i \leq 4).$$

  - Four one-bit $u_i$ are used to represent whether $Z_i$ is faulty ($u_i=0$) or not
    
    $$u_i = (1 \oplus z_{8i-7}) \land (1 \oplus z_{8i-6}) \land (\ldots) \land (1 \oplus z_{8i})$$

  - Only one byte fault is injected, only one $u_i=0$
    
    $$(1 - u_1) \lor (1 - u_2) \lor (1 - u_3) \lor (1 - u_4) = 1,$$
    $$u_i \lor u_j = 1, \quad 1 \leq i < j \leq 4$$
Step 3: Solver

- Combine the equation set of GOST with injected fault and use **solver** to recover the secret key.

- **CryptoMiniSAT v2.9.4**, support multiple solution output

- The PC that runs CryptoMiniSAT has the following configuration: **Intel Core i7-2640M, 2.80 GHZ, and 4G bytes memory**. The operating system is **64-bit Windows 7**.
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## Experiment Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>the number of fault injections</td>
</tr>
<tr>
<td>$V(N)$</td>
<td>the number of variables in equation set</td>
</tr>
<tr>
<td>$A(N)$</td>
<td>the number of ANF equations in equation set</td>
</tr>
<tr>
<td>$u(N)$</td>
<td>the size of the generated scripts</td>
</tr>
<tr>
<td>$t(N)$</td>
<td>the time complexity (seconds) required in solver</td>
</tr>
<tr>
<td>$\tau$</td>
<td>threshold of the time complexity (seconds) in a successful AFA</td>
</tr>
<tr>
<td>$\varphi(N,\tau)$</td>
<td>the success rate</td>
</tr>
<tr>
<td>$\lambda(N)$</td>
<td>the entropy of the secret key in Scenario 1</td>
</tr>
</tbody>
</table>
Results of Scenario 1

$4n$ random faults are injected into $R_i$, $i = \{24, 26, 28, 30\}$ of GOST ($n$ faults for each $i$, $N = 4n$).

![Graph](image1)

\[ \lambda(N) = 2^{12.2} \]

![Graph](image2)

\[ \lambda(N) = 2^{16.7} \]

$N=8$ faults are required to recover the master key, which is less than 64 in [Kim10].
Results of Scenario 2

2n random faults are injected into $R_i$, $i = \{30, 31\}$ of GOST ($n$ faults for each $i$, $N = 2n$).

64 faults to recover the 8 S-Boxes

<table>
<thead>
<tr>
<th>S-box</th>
<th>$a_1, a_2, \ldots, a_{64}$</th>
<th>S-box</th>
<th>$a_1, a_2, \ldots, a_{64}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S1$</td>
<td>0x3e4a983e4b4a3174</td>
<td>$S5$</td>
<td>0x0ab8873cec12349e</td>
</tr>
<tr>
<td>$S2$</td>
<td>0xf478c97494c208a6</td>
<td>$S6$</td>
<td>0x1dcda3679486e34</td>
</tr>
<tr>
<td>$S3$</td>
<td>0x6986bf52669eeec3c</td>
<td>$S7$</td>
<td>0xf5c8eb982aead2b2</td>
</tr>
<tr>
<td>$S4$</td>
<td>0x5802b282ac52f22e</td>
<td>$S8$</td>
<td>0x5c4a3b560aba85b6</td>
</tr>
</tbody>
</table>
Results of Scenario 3

9n random faults are injected into $R_i$, $i = \{23, 24, 25, 26, 27, 28, 29, 30, 31\}$ of GOST ($n$ faults for each $i$, $N = 9n$).

270 faults for the recovery of both of the key and 8 S-Boxes
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Conclusion of Our Work

Make a comprehensive study of AFA on GOST

• **AFA is Efficient**: when the whole design of GOST is known, the key recovery requires only 8 fault injection, less than 64 in previous DFA work.

• **AFA is Powerful**: can be used for reverse engineering, even both the key and S-Boxes are secret.

• **AFA is Automatic**: no need to analyze the fault propagation.

• **AFA is Generic**: apply to different attack scenarios.

• **One lesson**: keeping some components in a cipher secret cannot guarantee its security.
Thanks!

Q & A

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