Fault Detection Structures for the Montgomery Multiplication over Binary Extension Fields

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Outline

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- Previous Work
- Time Redundancy Based Fault Detection in Montgomery Multiplication
- Parity-Based Fault Detection in the Bit-Serial Montgomery Multiplication
- Discussion and Comparison
- Conclusions
Background

The binary extension field $\mathbb{GF}(2^m)$:

- contains $2^m$ elements
- is an extension of $\mathbb{GF}(2) = \{0,1\}$
- is associated with an irreducible polynomial

$$F(z) = z^m + f_{m-1}z^{m-1} + \cdots + f_1z + 1,$$

where $f_i \in \{0,1\}$ for $i = 1$ to $m-1$. 
Background

• Assuming $x$ is a root of $F(z)$, i.e., $F(x) = 0$, each element of $\mathbb{GF}(2^m)$ can be represented as a polynomial of degree $m - 1$.

$$A \in \mathbb{GF}(2^m) \iff A = a_{m-1}x^{m-1} + \cdots + a_1x + a_0,$$

where $a_i \in \{0, 1\}$ for $i = 0$ to $m - 1$.

This representation is called the polynomial basis representation.
Background

• Assuming $A$ and $B$ are two elements of the binary extension field and $r = x^m |$ the Montgomery factor satisfying

$$\gcd(r, F(x)) = 1$$

• The **Montgomery multiplication** over binary extension fields is defined as

$$C = A \cdot B \cdot r^{-1} \mod F(x),$$

$$r \cdot r^{-1} = 1 \mod F(x).$$
Previous Work

- Time redundancy based concurrent error detection scheme for semi-systolic implementation of the Montgomery multiplication algorithm

- Based on performing two different multiplications: the polynomial basis multiplication and the Montgomery multiplication
Previous Work

• Assuming $A, B \in \text{GF}(2^m)$, $\overline{A}$ and $\overline{B}$ are the Montgomery residues computed by $A \cdot x^m \mod F(x)$ and $B \cdot x^m \mod F(x)$ respectively, then

\[
C = A \cdot B \mod F(x)
\]

\[
\overline{C} = \overline{A} \cdot \overline{B} \cdot x^{-m} \mod F(x)
\]

\[
\overline{C} = \overline{A} \cdot \overline{B} \cdot x^{-m} \mod F(x) = (A \cdot x^m) \cdot (B \cdot x^m) \cdot x^{-m} \mod F(x) = A \cdot B \cdot x^m \mod F(x) = C \cdot x^m \mod F(x).
\]
Previous Work

• Error detection flowchart using the time redundancy

\[ A \cdot x^m \mod F(x) \mid B \cdot x^m \mod F(x) \]

\[ C = A \cdot B \mod F(x) \]

\[ \overline{C} = \overline{A} \cdot \overline{B} \cdot x^{-m} \mod F(x) \]

\[ \overline{C} = \overline{A} \cdot \overline{B} \cdot x^{-m} \mod F(x) = (A \cdot x^m) \cdot (B \cdot x^m) \cdot x^{-m} \mod F(x) \]

\[ = A \cdot B \cdot x^m \mod F(x) = C' \cdot x^m \mod F(x). \]

*By C.W. Chiou et al 2006*
• Now, we choose \( r = x^{m-1} \) as the Montgomery factor, so

\[
A' = A \cdot x^{-1} \mod F(x), \quad B' = B \cdot x^{-1} \mod F(x) = \sum_{i=0}^{m-1} b_i x^i
\]

• So we consider two Montgomery multiplications:

\[
C = A \cdot B \cdot x^{-m} \mod F(x), \quad C' = A' \cdot B' \cdot x^{-(m-1)} \mod F(x)
\]

\[
C'' = (A \cdot x^{-1}) \cdot (B \cdot x^{-1}) \cdot x^{-(m-1)} \mod F(x) = C \cdot x^{-1} \mod F(x)
\]
**The Modified flowchart**

\[ C' = A \cdot B \mod F(x) \]
\[ C' = A' \cdot B' \cdot x^{-(m-1)} \mod F(x) \]

\[ C'' = (A \cdot x^{-1}) \cdot (B \cdot x^{-1}) \cdot x^{-(m-1)} \mod F(x) = C \cdot x^{-1} \mod F(x) \]
The Montgomery multiplication can be implemented by using a semi-systolic architecture.

Using the new Montgomery factor, the latency of the architecture is $m_1$, equal to the latency of the polynomial basis multiplication.
New Parity-Based Fault Detection Scheme

Algorithm 1 Bit-level Montgomery multiplication over $\text{GF}(2^m)$

Inputs: $A, B, F(x)$
Output: $C = A \cdot B \cdot r^{-1} \mod F(x)$

Step 1: $T := 0$
Step 2: For $i := 0$ to $m - 1$
Step 3: $T' := T + b_i A$
Step 4: $T'' := T' + t_0 F(x)$
Step 5: $T := T'' / x$
Step 6: $C := T$

*By CK Koc and T Acar 1998*
Algorithm 1  Bit-level Montgomery multiplication over \( \text{GF}(2^m) \)

Inputs: \( A, B, F(x) \)
Output: \( C = A \cdot B \cdot r^{-1} \mod F(x) \)
Step 1: \( T := 0 \)
Step 2:  For \( i := 0 \) to \( m - 1 \)
Step 3: \( T' := T + b_i A \)
Step 4: \( T'' := T' + t' F(x) \)
Step 5: \( T := T'' / x \)
Step 6: \( C := T \)

the latency of \( m \) clock cycles

delay of \( 2(T_A + T_X) \)

\( 2m - 1 \) AND gates and \( 2m - 1 \) XOR
Lemma 1: The parity of $T^{(i)}$ equals $P_{T^{(i-1)}} + b_i \cdot P_A + t_{0}^{(i-1)} + b_i \cdot a_0$, where $P_{T^{(i-1)}}$ is the parity of $T^{(i-1)}$, $P_A$ is the parity of $A$, and $t_{0}^{(i-1)}$ is the LSB of $T^{(i-1)}$. 
\[ P_T^{(i-1)} + b_i \cdot P_A + t_{0}^{(i-1)} + b_i \cdot a_0 \]

delay of \( T_A + 2T_X \)

the latency of \( m \)
Discussion and Comparison

• The time redundancy based scheme:
  • The original scheme
    \[ H \cdot x^m \mod F(x). \]
  taking into account that
  \[ x^m = f_{m-1}x^{m-1} + \cdots + f_1 x + 1 \]
  We have
  \[ H \cdot x^m = (h_{m-1}x^{m-1} + \cdots + h_1 x + h_0) \cdot \]
  \( (f_{m-1}x^{m-1} + \cdots + f_1 x + 1) \mod F(x). \]

The area complexity of \( O(m^2) \)
The time complexity of \( O(\log_2 m) \)
Discussion and Comparison

- Time redundancy based scheme:
  - The modified scheme

\[
H \cdot x^{-1} = (h_{m-1}x^{m-1} + \cdots + h_1x + h_0) \cdot x^{-1} \mod F(x),
\]

taking into account that

\[
x^{-1} = x^{m-1} + f_{m-1}x^{m-2} \cdots + f_1,
\]

We have

\[
H \cdot x^{-1} = (h_0)x^{m-1} + (h_0f_{m-1} + h_{m-1})x^{m-2} + \cdots + (h_0 \cdot f_1 + h_1)
\]

The area complexity of \( O(m) \)

The constant time complexity of \( T_A + T_X \)
Discussion and Comparison

• Time parity based scheme:
  • The critical path delay $T_A + 2T_X$
  • The latency of $m$
    ➢ Concurrent parity prediction
  • Three XOR gate and two AND gates (Constant)
  • Final XOR tree
    • The time complexity of $\left\lceil \log_2 (m + 1) \right\rceil \cdot T_X$
    • The area complexity of $m$ XOR gates
Conclusions

- Two error detection schemes have been introduced:
  - Modification of an existing time redundancy based scheme for semi-systolic implementation of the Montgomery multiplication.
  - A new parity based scheme for the bit-serial Montgomery
- The time and complexity of the previous time redundancy based scheme is significantly improved. While it has the same error detection capability.
- The parity based scheme is capable of obtaining the parity of the intermediate and the final result without any time overhead and with a constant hardware overhead.
Thanks!